

## **Menautkan Bukti Matematika dalam Konstruktivisme**

### ***Contextualising Mathematical Proof within Constructivism***

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#### **ABSTRAK**

Pemahaman tentang konstruktivisme sebenarnya belum utuh jika belum dapat diterapkan dalam praktek profesional. Dengan memperhatikan filsafat pendidikan matematika terkini, bisa ditelusuri suasana yang mungkin dan diwujudkan dengan menggunakan konstruktivisme sebagai perantara. Perantara ini digunakan untuk menciptakan tautan yang di dalamnya proses belajar mengajar tentang bukti matematika dapat dilaksanakan.

Kata kunci: Konstruktivisme, bukti matematika, belajar mengajar.

#### **ABSTRACT**

An understanding of constructivism would be complete when it could be implemented within professional practice. Taking into account the current philosophies in mathematics education, I can see a possible setting which can be generated by using constructivism as a mediator. This mediator is used to set the context in which the processes of teaching and learning of mathematical proof are conducted.

Key words: Constructivism, mathematical proof, teaching and learning.

#### **INTRODUCTION**

An understanding of constructivism would not be complete when we could not find (or construct) the way through which it can be implemented in our professional practice. Actually, by using constructivism as referent within an educational research project, it has been proven that, to some extent, the real implementation of it is

evident as it helped to explain some dimensions of the findings of the research. Blending all the constructed understanding and experiences in conducting the project, this article will synthesize a discussion covering the topic of mathematical proof and how it could be contextualised within the perspective of constructivism.

The process of learning and teaching of mathematical proof, based on

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my experience, has been proved to be dominated by the deductive axiomatic system. Actually, this phenomenon will not become a problem when all learners could succeed in their learning contextualised in this system. However, the facts have shown that students encounter various difficulties in their learning. The domination of the deductive axiomatic system with its criteria of rigorous proof is not to be replaced by the inductive approach as these two sides of mathematics have their own space and roles. All we have to do is to mediate them so that they could function together. Instead of perpetuating an irreducible divergence between the two sides, one should exist as complement to another. The constructivist perspective seems to be a potential mediator. As a theory of knowing, it should be capable of explaining the phenomena of learning.

An informed conception of mathematical proof should include a consideration of the roles of proof (Knuth, 2002b). In the process of teaching and learning of mathematical proof, students should be exposed to various roles of proof. However, the traditional models of learning activities seem not to be able to facilitate such a learning demand. As a consequence, students have limited conceptions of the roles of proof. Learning activities very much depend on the epistemology we implement in the process of teaching and learning. Constructivist perspective seems to be capable of facilitating learning process that enables the roles of proof to be performed appropriately.

The focus of the following discussion will be twofold. The first focus is the potential link between the deductive

axiomatic system and the constructivist referent. Having this link developed, the discussion will move to the second focus, that is, addressing the roles of mathematical proof by using the various forms of constructivism. This second part will discuss the relationship between the roles of proof and the constructivist perspectives, especially about the forms of constructivism that can facilitate the implementation of the roles of proof. This article will be concluded with a discussion of the implication of those relationships identified. However, prior to presenting further explanation, the philosophy in mathematical education, the deductive axiomatic system, and constructivism as referent in order to provide a base for further discussion will be briefly described.

### **COMPETING PHILOSOPHIES IN MATHEMATICS EDUCATION**

In this section, I discuss the prevailing philosophies that compete within the context of mathematics education. After a general description, a discussion of the deductive-axiomatic system which is contextualised within the infallible-absolutist philosophy is presented and then it is followed by an analysis of constructivist referent.

In general, there are two extreme classes of philosophy occupying the world of mathematics. In simple terminology, those two classes could be classified as infallible-absolutist and fallible-constructivist. Ernest (1991), has discussed them almost thoroughly within two contexts: education and society, where mathematics is bestowed a level of significance higher than that in the other contexts. Within the school of infallible-

absolutism, according to Ernest (1991), mathematics is objective knowledge with no social responsibilities assumed. In this sense, learners experience cultural alienation from mathematics of which the relationship of mathematics with human affairs is deliberately excluded. Further, human affairs are considered as having no relevance to mathematics.

On the contrary, within the school of fallible-constructivism, mathematics is considered as a process of inquiry and coming to know (Ernest, 1991). Teaching and learning mathematics are conducted to empower the learners so that they could create their own mathematics. The kind of mathematics, however, is not seen as a final product, but rather, it needs and will experience a continuous and sustained process of reshaping, development, adaptation, and refinement based upon the context of the lives of the learners in order for the knowledge to present its significance for them. Mathematics acts more as a subjective knowledge. Simply put, learners are mathematically empowered and emancipated.

#### **DEDUCTIVE-AXIOMATIC SYSTEM**

From the perspective of the epistemology of the Old Humanist Mathematicians in which the philosophy of absolutism and relativism are separated, conservative mathematicians create a community within which rigour of proof and purity of mathematics are perpetuated (Leigh-Lancaster, 2002; Ernest, 1991). Rigour and purity are set up in deductive axiomatic system. In terms of proof in teaching and learning mathematics context, absolutists claim that “without complete,

correct proof, there is no mathematics” and they consider mathematics as a system of absolute truth (Hersh, 1993).

Proof which is derived via deductive logic is a means of establishing the certainty of mathematical knowledge, but that certainty is not the absolute one (Ernest, 2003). It is clear that in doing mathematics, a set of assumptions is always taken as a base and there is no way of escaping them. Further, Ernest (2003) argues that all we can do is to minimise the set of axioms and rules of proof, and this reduced set should be completed with assumptions so that the mathematical theorems that follow form an assumed basis which results in non-absolute certainty. Although the discussion of deductive-axiomatic method does not provide much information about mathematical proof, this special method is the only one that could guarantee the truth of a mathematical assertion (Rota, 1996). However, the notion of rigour, according to Hanna (2000b), is another matter and she argues that the criteria of rigour do not secure the acceptance of a proof within the community of mathematicians if it does not promote and help the construction of understanding.

#### **CONSTRUCTIVISM AS REFERENT**

Regarding the learning and teaching process, constructivism is an alternative epistemology or theory of knowing which breaks the epistemological tradition in philosophy in which knowledge should represent a real world that is thought of as existing, separated from and independent of the knower (von Glasersfeld, 1995). From constructivist perspective, science is no

longer seen as a collection of facts to learn nor is mathematics a set of formulas to memorize (Treagust et al., 1996a). It emphasizes the importance of active learners and teachers in their engagement in the process of constructing knowledge or understanding (von Glasersfeld, 1990), which perpetuates the ethics of emancipation and care (Taylor, 1998). Even though it is open to different interpretations (von Glasersfeld, 1995), which allows a range of different views about it, constructivism has undoubtedly been a productive influence in research on students' and teachers' conceptions and in teaching and learning science and mathematics (Steffe and Gale, 1995).

As a theory of knowing, according to von Glasersfeld (1990), constructivism holds central principles, that is: knowledge is actively built up by a learner and received either through the senses or by way of communication; cognition is adaptive in order to be viable; and the organization of the experiential world of a learner is supported by the cognition. However, there is not enough evidence that the discovery of an objective ontological reality is facilitated by the cognition (von Glasersfeld, 1990).

According to constructivism, it is impossible to literally transfer ideas into students' heads intact; rather, the meanings from the words or visual images are constructed by them. Consequently, when engaging in this construction of meaning, what the learner already knows, that is, prior knowledge or understanding, is of central importance (Treagust et al., 1996a). Tobin and Tippins (1993) state that the optimum benefits of constructivist

epistemology could be acquired when we use it as a referent, that is, as a way to make sense and understand what we see, think, experience, and do. Constructivism, in a clear contrast position to absolutism, does not refer to the truth of knowledge. Rather, it applies the criteria of viability upon which knowledge or understanding is justified according to its usefulness for the owner (von Glasersfeld, 1995).

Geelan (1997), has identified six forms of constructivism, that is, personal constructivism, radical constructivism, social constructivism, social constructionism, critical constructivism, and contextual constructivism. Each form should not be seen as a set of method or a set of beliefs, because each is defined loosely by a collection of writings of particular individual (Dougiamas, 1998). According to Geelan (1997), "there is not, and should not be, 'One True Way' in constructivism – a variety of perspectives is both more flexible and more powerful". The six forms actually construct a coherent epistemological framework.

## **MATHEMATICAL PROOF**

What is a proof? Various answers might emerge for this question depending on the context in which it is asked. Within the context of mathematics, proof has its own uniqueness which makes it different from proofs in other disciplines.

The concept of proof that has become fundamental in mathematical endeavours since ancient times (Lee, 2002), is indeed the very essence of and is very important in mathematics (Markel, 1994). It makes mathematics with its uniqueness different from all other disciplines and

areas of human activity (Reid, 2001; Hanna 2002b). Knuth (2002a), defines a proof as a deductive argument that explains the truth of a statement by utilizing other mathematical results and/or insight into the mathematical structure involved in the statement. Proof, as an educational realm in mathematics, is rather difficult to put a finger on because it involves questions about formality, rigour, and logic, and at the same time, especially in undergraduate mathematics study, it deals with persuasion, meaning, generalisation, generating ideas to solve a given problem, and the handling of complicated structures (Mamona-Downs and Downs, 2002).

Looking critically at the situation in the department of mathematics where I work, it was found that the criteria of rigour play an influential role in teaching, especially within pure mathematical units such as Real Analysis and Abstract Algebra, in which lessons revolve around the activity of conjecturing and proving. Further, the definition-theorem-proof sequence was normally implemented in the lesson, and such a situation, according to Mamona-Downs and Downs (2002), sometimes leads students to see a formal proof as a game of meaningless manipulation of symbol which eventually results in their difficulties when they encounter even a simple proof.

### **POSSIBLE LINK?**

This section discusses the potential relationship which results from a synthesis between the deductive-axiomatic system and the perspective of constructivism in order to have a balanced view of the notion of proof. It is argued that there is an

interesting link between proof, which is constructed within the context of a deductive-axiomatic system with its potential embedded non-absolute certainty, and constructivism with its notion of viability.

Mathematics is developed upon agreements among mathematicians within their community. Axioms and postulates as the bases of the deductive-axiomatic system are derived from consensus among them. This system is considered as the framework that ensures the so-called absolute truth of mathematics. However, from this point, it can be implied that the absolute truth is actually relative to the community itself, and the sense of non-absolute nature consequently must be embedded in.

No matter how much an absolutist view claims mathematics as universal, objective and certain, with its truths being discovered through the intuition of mathematicians and then being established by proof (Ernest, 1999), its universality, objectiveness and certainty are still confined within the community itself. When someone is said to be possessing mathematical ideas, it means that they are contributing and participating in the truths of mathematics; that they are involved in the greater shared reality (Threlfall, 1996). Proof via deductive logic, which is considered as a means for establishing the certainty of mathematics, is something we cannot completely rely upon (Ernest, 2003) because Lakatos (1978) has shown that despite all the foundational works and development of mathematical logic, the quest for mathematical certainty leads unavoidably to an infinite regress.

From such a critical point, Ernest (1999) argues that mathematics is actually fallible and it is created by mathematicians who dedicate themselves to both formulating and critiquing new knowledge in a formal conversation before it counts as accepted mathematics. A mathematical idea is first created by a person (mathematician) and then it is judged within the community. Social interactions among members that take place in order to discuss, scrutinise, and justify that idea are the essence of constructivism. Further, according to (social) constructivism, mathematics occupies an intermediate position because it is above the status of only a collection of subjective beliefs and is below the position as a body of absolute objective knowledge (Ernest, 1999).

Given the argument of Rota (1996), regarding proof as the only means to ensure the mathematical assertion's truth and that its absolute nature actually carries along certain non-absolute nature, the notion of viability (von Glasersfeld, 1995) could be put forward as a complement to the already existing criteria. This alternative criterion, which is embraced within constructivism, more concerns usefulness. In accordance with this criterion, Hanna (2000b), has argued that the notion of rigour only is secondary in importance to understanding, and the legitimation of a proof in mathematics is gained from its significance (usefulness) in promoting understanding.

Considering mathematics as cultural knowledge with a moderate status (Ernest, 1999) will enable students to learn more contextualised mathematics. From a critical constructivist perspective (Taylor, 1996), teaching and learning for conceptual

understanding of mathematical proof could be turned into an enjoyable activity when proof is positioned as an emancipatory agent for empowering students. Students will also be freed from the repressive myths of cold reason and hard control (Taylor, 1996), which is likely to be exerted by the strong fanatics who have an attachment to the rigorous criteria of a deductive-axiomatic system, when teaching and learning process takes into account various perspectives facilitated by the fallibilist philosophy of mathematics. However, this is not to advocate the weakening of the community standards of proof, as Hanna (1995) reminds us, but rather, to provide the knowledge learnt and constructed with more characteristics of utility and functionality (Moslehian, 2003).

## **ADDRESSING THE ROLES OF PROOF**

Learning and teaching is expected to reflect all of the roles of proof. It is useful to consider a whole range of roles proof plays in mathematical practice (Hanna, 2000a), and put proportional emphasis on each role especially in the course of learning and teaching or instruction (Hersh, 1993). There have been many studies conducted on various possible roles proof can play in learning and teaching mathematics. Some roles that proof could play in learning and teaching mathematics are the roles of verification, explanation, systematisation, discovery /invention, communication, exploration, construction, incorporation, aesthetic, and personal self-realisation (Hanna, 2002a, 2002b; Knuth, 2002a, 2002b; de Villiers, 1990; Hanna, 1990; Ernest, 1991; Lakatos, 1976; Hanna and Jahnke, 1996). Hanna

(1995), argues that while in mathematical practice the main function of proof is verification and justification, its main role in mathematics education actually is explanation.

Some of those roles seem to be indistinguishable and overlap to one another (Knuth, 2002b). Also, some of them exist in close relationship to other functions such as: incorporation and systematization, or discovery and exploration. In order to systematize the discussion, I will divide those ten roles into four groups. This grouping is attempted by considering the commonalities evident among those roles. The first group covers the roles of verification, explanation, and communication. This group deals with demonstrating, explaining, or communicating the truth of a mathematical statement. The second group covers the roles of systematization and incorporation. This group pertains to the notion of organizing mathematical ideas. The third group comprises the roles of discover, exploration, and construction. This group deals with the creation of mathematical results or theory. The last group encompasses the roles of aesthetic and personal self-realisation. This group refers to personal artistic dimension of proof. Further, instead of explaining each of the roles, I will explain them by their groups; while at the same time I will analyse them in the context of constructivist perspectives.

In order to analyse the roles of proof, the six forms of constructivism proposed by Geelan (1997) are employed. Each group of proof roles will be discussed by using related faces of constructivism;

however, certain forms might be emphasised more than the others when necessary. The following is the discussion of the relationship between constructivism perspectives and the roles of proof. The discussion presents what and how constructivist perspectives can facilitate the performance of the roles of proof. Following the discussion is the explanation of the benefits which can be gained from identifying those potential relationships.

### **1. The First Group**

Several forms of constructivism can be used to explain this group. Hanna (2000a), states that apparently every student enters the world of mathematical enterprise with understanding of just the fundamental functions of proof, that is, verification and explanation. Verification is considered as the main and the most traditional role of proof in the course of mathematics (Hanna, 1983; de Villiers, 1990) which is sometimes juxtaposed with the notion of conviction and validity as the psychological personal aspect and social aspect of proof, respectively (Segal, 2000).

From this point, the notion of personal constructivism can be used to analyse those two roles. Students do not passively receive the truth of a mathematical statement, but rather, they actively construct their understanding by proving it in order to verify its truth. However, the demonstrative purpose of proof seems to be not enough for the learners because the knowledge constructed might be superficial in nature. Proof is to answer the essential question of why? (Hanna, 2000a; Hanna, 2000b). Accordingly, explanative function of proof

is distinguished from its demonstrative function by mathematicians (Steiner, 1978). Some proofs are more explanative than other (Hanna, 2000b) which help mathematicians understand why a statement is true (Hanna, 1995; Hersh, 1993). They provide an explanatory power and personal meaning which is sometimes absent in classroom instruction (Schoenfeld, 1994). This explanatory function of proof helps students forge and strengthen their understanding. Within the context of radical constructivism, this refers to its second principle, that is, to make the experience more meaningful. In this sense, student understanding will be more meaningful because they not only know that a mathematical statement is true, but they also know why it is true.

The communicative function of proof, to some extent, refers to the transmission of mathematical knowledge (Hanna and Jahnke, 1996). Focusing on this side might lead us to an impression that this role does not work in accordance with the constructivist perspective. However, from the other side of this communicative role, it can be considered as the way through which students can express their understanding. Expressing their constructed knowledge has two goals, that is, to scrutinise their understanding and to express their idea. To scrutinise their understanding necessitates social interactions among all parties involved in learning and teaching activities. This role, to a great extent, has a close relationship to the social constructivism. Students will be given voice. On the other hand, communicating their understanding (proof they have constructed) means that the

students are given voice and opportunity to express themselves. These processes of empowering students with all their potentials can take place within context of critical constructivism.

There seems to be no significant problem in functioning the roles of proof contained within this group. Researchers have identified that the verification and explanation functions are the most widely known by mathematical learners (for example, see Knuth, 2002a; 2002b). I am arguing that these two roles can be considered as the “trivial” roles of proof. Students mostly are exposed to these roles because they are implemented and performed most frequently among the other roles.

## **2. The Second Group**

The role of proof in systematizing the results of mathematical activities into a deductive system of definition, axioms, postulates, and theorems is probably its most mathematical function in nature (Knuth, 2002a). Meanwhile, Hanna and Jahnke (1996), state that incorporation is to organise a well-known fact into a new framework which will provide a space for a fresh perspective toward the system. Incorporation also holds the meaning of setting the fact into an alternative framework.

Knuth (2002a), surmises that many students view the theorems they are assigned to prove as merely independent of one another rather than being structured in a deductive axiomatic system. In my practice, I found students needed to be constantly reminded about the previous theorems they have learned in order to help



them prove the new ones. I think this problem can be overcome when the notion of social constructionism is taken into account in the process of teaching and learning mathematical proof. Within the context of social constructionism, knowledge is considered to reside within the societies “that the consensus processes of language-use and meaning-making are social in nature” (Geelan, 1997).

Although Gergen (1995), proposes this form of constructivism within the domain of language, it can be expanded into the domain of mathematical proof. His suggestions concerning the language-use and the meaning-making also apply in the domain of mathematical proof. The process of systematization, which is played by proof, is very much context dependence. The language used within the deductive axiomatic system as the context only serves the community of mathematicians. The meanings prevailing in the system, for example, the terminology used in definition, axioms, and postulates, are achieved through social interdependence and agreement among mathematicians involved.

These two roles actually can be seen from two aspects, that is, the individual and social aspects. With their social aspect, the systematization and incorporation take place within the community of mathematicians. They are implemented to organise the mathematical results within the mathematics which is perpetuated in the community. With their individual aspect, these two roles were functioned in organising the mathematics which is part of the knowledge constructed by a learner. This second aspect can be

contextualised within the radical constructivism, especially with its second principle. Proof can play the role of systematising and incorporating the new knowledge constructed by an individual into his/her existing knowledge. To some extent, the process is parallel with the notion of assimilation and accommodation proposed by Piaget. Thus, the notion of personal constructivism which based upon Piaget’s proposal also could become the context within which the individual aspect of systematization and incorporation function of proof can be performed.

### **3. The Third Group**

To a great extent, proof as a means of discovery is closely related to its explorative function. Knuth (2002a), says that discoveries could be started from firstly generating a conjecture which could be done through exploration; and then verifying it using deductive proof. To this point, proof is used to create a new mathematical result. Exploration refers more to the further study or elaboration of definition and finding the whole meaning it holds. According to Lakatos (1976), there should be a careful elaboration of definition especially when we examine theorems derived from it. Proof is a means of exploration in a way that the existing theorems which have been proven could lead to the invention of new mathematical ideas such as theorems or lemmas; and thus enrichment is taking place. Exploration is also filled with the process of rethinking, reflecting, refining, and perfecting, in order to create a better mathematical enterprise. Proof also plays an important role in the reconstruction of any mathematical entity.

This role will function optimally within the epistemological and ontological context in which mathematics is viewed as a result of social construction (Ernest, 1991). The product of mathematics is not the final one; construction and reconstruction are still taking place.

Perhaps the most salient relationship between the roles of proof and the constructivist perspectives can be found within this third group. It is because the construction, exploration, and discovery, to a great extent, characterise the perspective of constructivism. From personal constructivism, to radical constructivism, up to contextual constructivism, the main idea is that learners construct their own knowledge. Within the constructivist-based process of teaching and learning, students are facilitated to do exploration, whether it is done individually or within group. All six forms of constructivism can be used to explain the roles included in this group. Simply put, these two roles of proof should be easier to be performed within the process of learning and teaching of mathematical proof.

Teaching and learning mathematical proof within the context of constructivism can implement problem solving approach. The roles of proof in exploration, discovery, and construction can be performed well in problem solving activity. Students are assigned to solve a problem which requires them to do discovery endeavour, exploration, and construction. The notion of viability perpetuated within constructivism with its criteria of usefulness also enables the implementation of these three roles. Students are facilitated to explore various solutions to the problem.

However, this does not mean that all solutions found must be considered equally desirable, because the degree of viability will then be justified by referring to some other scale of values such as speed, economy, convention, and elegance (von Glasersfeld, 1995).

#### **4. The Fourth Group**

A cursory observation shows that the characteristics of the beauty of mathematics are different from those of artistic beauty (Rota, 1996). The beauty of a proof shows when “it gives away the secret of theorem, when it leads us to perceive the inevitability of the statement being proved” (Rota, 1996). Mathematicians have a distinctive sense of beauty. They strive not only to construct irrefutable proofs but also to present them in a clear and compelling fashion dictated by an aesthetic sense and logic. Regarding the personal self-realisation, examples can be taken from the success of mathematicians to construct a proof for a certain theorem. The success of Andrew Wiles in constructing the proof of Fermat’s Last Theorem (Stewart, 1999) has provided him with the great satisfaction of acknowledgement from the community of mathematicians. Alibert and Thomas (1991) claim that constructing conjectures and proof have an intimate aspect through which a mathematician gains contentment and self-realisation. de Villiers (1990) argues that the production of proof is a kind of testing ground for the stamina, creativity, and inventiveness of a mathematician.

Students can be facilitated to learn mathematical proof implementing the roles

of aesthetics and personal-self actualisation within the context of learning which is free from tight control. The presence of the repressive myths of cold reason and hard control will confine student learning and it results not only in disability to see the beauty of mathematics through proof but also in disability to express their self-actualisation through constructing proof. Within the context of critical constructivism, students are freed from repressive myths and they are also emancipated and cared. It means that they are empowered and given a voice to express themselves. Thus, in order to perform the aesthetics and the personal self-realisation roles of proof, the process of teaching and learning could be contextualised within the perspective of critical constructivism.

## **REFLECTION**

From the elaboration above, it is found that each role of proof can be contextualised within one or more face of constructivism. It means that in order to facilitate the optimal implementation of the roles of proof, various forms of constructivism should be employed. This is in accordance with the idea of Geelan (1997) that holding or implementing those forms of constructivism will become more powerful and flexible when they are in concert with one another.

Having the relationships between the roles of proof and the constructivist perspectives and the possible contextualisation of the roles of proof within constructivist perspectives identified, the process of teaching and learning of mathematical proof can be

improved. Any endeavour to implement the constructivist epistemology in the practice should aim at devising suitable learning activities and implementing them by using appropriate teaching strategies. Of course, the learning activities and teaching strategies intended are those that developed under the light of constructivism. Various constructivist teaching strategies which were developed within systematic research can be found within Treagust et al. (1996b).

It is important to be aware that the implementation of constructivism in classrooms is neither widespread nor systemic (Airasian and Walsh, 1997). Regarding the teaching strategies developed under the light of constructivism, researchers admit that to present adequate evidence of the success of constructivist teaching approaches is somewhat difficult (Duit and Confrey, 1996). The problem is in condensing promising results into measures that enable critical comparison with the traditional methods. However, some research results that allow comparisons signify that constructivist approaches, which concern understanding mathematical contents, are mostly better than the traditional ones (Duit and Confrey, 1996). The case of teaching and learning of mathematical proof is still in question and it necessitates systematic studies to provide the answers.

The following cautions provided by Airasian and Walsh (1997) are worth considering. They remind that:

- We should be able to recognize the difference between an epistemology of learning and a well-thought-out and manageable instructional approach for implementing it,

- We should not fall into the trap of believing that constructivist instructional techniques provide the sole means by which students construct meanings,
- We should not assume that a constructivist orientation will make the same demands on teaching time as a non-constructivist orientation, and
- We should not believe that the opposite of “one-right-answer” reductionism is “anything-goes” constructivism.

As a theory of knowing, we should bear in mind that constructivism is one of various theories of knowing available. The multiplicity of theoretical frameworks existing within the discipline of mathematics should be maintained because all perspectives have value, whether consistent with other views or not (Geelan, 1997). They are not to be compared or discarded, they are to be utilised, applied, argued, articulated, and criticised (Boursfeld, 1988). With special referent to mathematics, von Glasersfeld (1990) states that rote learning and the focus on adequate performance are not to be discarded completely from constructively oriented instruction. All we should do is to select which learning epistemology or perspective is suitable for us by taking into account considerations such as: the context we are in and the outcomes we aimed at.

### **CONCLUDING REMARKS**

The process of teaching and learning of mathematical proof becomes problematic because the topic of proof is still found to be difficult for students to understand. Research findings show that students’ conceptions of mathematical

proof are still limited. Mathematical proof which is basically contextualised within the deductive axiomatic system with its criteria of rigour has potential link with the constructivist perspective with its notion of viability. Given such a link, teaching and learning of mathematical proof which implement various roles of proof can be contextualised within constructivist epistemology. The many faces of constructivism are potential to address the roles of proof and to create teaching and learning context within which the proof can play its roles significantly. It depends on the teachers and learners to decide whether they implement constructivism within the process of teaching and learning. By taking constructivist perspective, teaching and learning activities can be devised within which the roles of proof can be performed.

My understanding is something growing dynamically and is undergoing a sustained development. The dynamic and sustained growth and development of my understanding or knowledge about constructivism is to ensure that the knowledge or understanding I construct is viable and can help me increase my ability to survive.

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## REFERENCES

- Airasian P and Walsh ME. 1997. *Constructivist Cautions*. Phi Delta Kappan, 78, 324-326.
- Alibert D and Thomas M. 1991. Research on Mathematical Proof. In Tall, DO. (Ed.). *Advanced Mathematical Thinking* (pp. 215-230). The Netherlands: Kluwer Academic Publishers.
- Bauersfeld H. 1988. Interaction, Construction and Knowledge: Alternative Perspectives for Mathematics Education. In Grouws, DA, TJ Cooney, and D Jones. (Eds.). *Perspectives on Research on Effective Mathematics Teaching* (Vol. 1, pp. 27-46). Reston, VA: Lawrence Erlbaum Associates.
- de Villiers M. 1990. *The Role and Function of Proof in Mathematics*. Pythagoras, 24, 17-24.
- Dougiamas M. 1998. *A Journey into Constructivism*. <http://dougiamas.com/writing/constructivism.html>. Accessed on September 13, 2003.
- Duit R and Confrey J. 1996. Reorganising the Curriculum and Teaching to Improve Learning in Science and Mathematics. In Treagust, DF, R Duit, and BJ Fraser. (Eds.). *Improving Teaching and Learning in Science and Mathematics* (pp. 79-93). New York: Teacher College Press.
- Ernest P. 1991. *The philosophy of Mathematics Education*. London: The Falmer Press.
- Ernest P. 1999. *Is Mathematics Discovered or Invented?* Philosophy of Mathematics Education Journal, 12. <http://www.ex.ac.uk/~PERnest/pome12/article2.htm>. Accessed on October 18, 2003.
- Ernest P. 2003. *Social Constructivism as a Philosophy of Mathematics: Radical Constructivism Rehabilitated?* <http://www.ex.ac.uk/~PERnest/soccon.htm>. Accessed on October 17, 2003.
- Geelan DR. 1997. *Epistemological Anarchy and the many Forms of Constructivism*. Science and Education, 6, 15-28.
- Gergen KJ. 1995. Social Construction and the Educational Process. In Steffe, LP and J Gale. (Eds.). *Constructivism in Education* (pp 17-39). Hillsdale, NJ: Lawrence Erlbaum.
- Hanna G. 1983. *Rigorous Proof in Mathematics*. Toronto, Ontario: OISE Press.
- Hanna G. 1990. *Some Pedagogical Aspects of Proof*. Interchange, 21(1), 6-13.
- Hanna G. 1995. *Challenge to the Importance of Proof*. For the Learning of Mathematics, 15(3), 42-49.
- Hanna G. 2000a. *A Critical Examination of Three Factors in the Decline of Proof*. Interchange, 31(1), 21-33.
- Hanna G. 2000b. *Proof, Explanation and Exploration: An Overview*. Educational Studies in Mathematics, 44, 5-23.
- Hanna G and Jahnke HN. 1996. Proof and Proving. In Bishop, AJ, K Clements, C Keitel, J Kilpatrick, and C Laborde. (Eds.). *International Handbook of Mathematics Education* (Part 2, pp. 877-908). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Hersh R. 1993. *Proving is Convincing and Explaining*. Educational Studies in Mathematics, 24, 389-399.
- Knuth EJ. 2002a. *Secondary School Mathematics Teachers' Conceptions of Proof*. Journal for Research in Mathematics Education, 33(5), 379-405.
- Knuth EJ. 2002b. *Teachers' Conceptions of Proof in the Context of Secondary School Mathematics*. Journal of Mathematics Teacher Education, 5, 61-88.
- Lakatos I. 1976. *Proof and Refutations*. Cambridge: Cambridge University Press.

- Lakatos I. 1978. Infinite Regress and Foundations of Mathematics. In Worrall J and Currie G. (Eds.). *Imre Lakatos, Philosophical Papers: Mathematics, Science and Epistemology* (Vol. 2, pp. 3-23). Cambridge, England: Cambridge University Press.
- Lee JK. 2002. *Philosophical Perspective on Proof in Mathematics Education*. Philosophy of Mathematics Education Journal, 16. <http://www.ex.ac.uk/~PERnest/pome16/docs/lee.pdf>. Accessed on May 14, 2003.
- Lancaster LD. 2002. *System, Values and Mathematics Education*. Paper presented at The International Conference of Values in Mathematics and Science Education 2002 at Monash University, Victoria, Australia, October 3-5, 2002.
- Downs MJ and Downs M. 2002. Advance Mathematical Thinking with a Special Reference to Reflection on Mathematical Structure. In English, LD. (Ed.). *Handbook of International Research on Mathematics Education* (pp. 165-195). Mahwah, NJ: Lawrence Erlbaum Associates.
- Markel WD. 1994. *The Role of Proof in Mathematics Education*. School Science and Mathematics, 94(6), 291-295.
- Moslehian MS. 2003. *A Glance at Postmodern Pedagogy of Mathematics*. Philosophy of Mathematics Education Journal, 17. <http://www.ex.ac.uk/~PERnest/pome17/pomoped.htm>. Accessed on Retrieved October 18, 2003.
- Reid D. 2001. *Proof, Proofs, Proving and Probing: Research Related to Proof*. Paper based on a Short Oral Presentation at the Twenty-Fifth Annual Conference of the International Group for the Psychology of Mathematics Education, Utrecht, The Netherlands, July 12-17, 2001.
- Rota G. 1996. *Indiscrete Thoughts*. Boston: Birkhauser.
- Schoenfeld A. 1994. *What do We Know about Mathematics Curricula?* Journal of Mathematics Behavior, 78(6), 448-456.
- Segal J. 2000. *Learning about Mathematical Proof: Conviction and Validity*. Journal of Mathematical Behavior, 18(2), 191-210.
- Steffe L and Gale J. (Eds.). 1995. *Constructivism in Education*. Hillsdale, NJ: Erlbaum.
- Steiner M. 1978. *Mathematical Explanation*. Philosophical Studies, 34, 135-151.
- Stewart I. 1999. *Proof and Beauty: Mathematical Proofs Used to be Short and Sweet, these Days They Read more Like War and Peace, or Worse, a Phone Directory. Is the Elegant Proof a Lost Art?* New Scientist, 162(2192), 29-32.
- Taylor PC. 1996. *Mythmaking and Mythbreaking in the Mathematics Classroom*. Educational Studies in Mathematics, 31, 151-173.
- Taylor PC. 1998. Constructivism: Value Added. In Fraser, BJ and KG Tobin. (Eds.). *International Handbook of Science Education* (pp. 1111-1123). Dordrecht, The Netherlands: Kluwer Academic.
- Threlfall J. 1996. *Absolutism or not Absolutism - What Difference does It Make?* Philosophy of Mathematics Education Newsletter, 9. <http://www.ex.ac.uk/~PERnest/pome/pompart6.htm>. Accessed on October 18, 2003.
- Tobin K and Tippins D. 1993. Constructivism as a Referent for Teaching and Learning. In Tobin, K. (Ed.). *The Practice of Constructivism in Science Education* (pp. 3-21). Washington: AAAS Press.

- Treagust DF, Duit R, Fraser BJ. 1996a. Overview: Research on Students' Preinstructional Conceptions - the Driving Force for Improving Teaching and Learning in Science and Mathematics. In Treagust, DF, R Duit, and Fraser BJ. (Eds.). *Improving Teaching and Learning in Science and Mathematics* (pp. 1-14). New York: Teacher College Press.
- Treagust DF, Duit R, Fraser BJ. (Eds.). 1996b. *Improving Teaching and Learning in Science and Mathematics*. New York: Teacher College Press.
- von Glasersfeld E. 1990. An Exposition of Constructivism: Why some Like It Radical. In Davis RB, CA Maher, and N Noddings. (Eds.). *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 19-29). Reston, VA: NCTM.
- von Glasersfeld E. 1995. A Constructivist Approach to Teaching. In Steffe L and J Gale. (Eds.). *Constructivism in Education*, (pp. 3-16). New Jersey: Lawrence Erlbaum Associates, Inc.