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Profiles of Students' Geometrical Thinking Levels based on Their Spatial Abilities

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Abstract
This research investigated students' geometrical thinking based on van Hiele's Geometrical Thinking levels for analysis, visualization, and abstraction according to students' spatial abilities. The research type was a case study. The research subjects were six grade XI students, chosen based on their spatial ability levels, namely, high, average, and low. Each level consisted of two subjects. Data collection was conducted using a spatial ability test and interview schedule, which the researcher developed from solid figure geometrical subject matters. Research's findings are (1) for high spatial-ability subjects, show an average score at the analysis level by identifying two properties of a spatial figure, (2) for average-level spatial-ability subjects, show the highest score at the analysis level by identifying several properties of a spatial figure. (3) Low spatial-ability subjects also show an average score at the analysis level by identifying two or three properties of the spatial figure.

Keywords: *Van Hiele's Geometrical Thinking Stages; Spatial Ability; Solid Figure.*

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INTRODUCTION

Geometry has been taught in schools from elementary to higher education. However, although students have received material on geometry since elementary school, not many of them can understand geometry from a practical point of view. The problem is teacher teaches geometry only by guessing without directly showing the real and how to measure geometric elements. In addition, teachers do not appreciate students who have special abilities with spatial/spatial abilities. Only those who are able to work on mathematical model problems with a number of formulas are considered to understand geometry.
This is the concern of the National Council of Teachers of Mathematics (NCTM), seeing the face of education which is increasingly rich in theory than practice. Therefore, the NCTM recommends that teaching geometry incorporates three-dimensional geometry and provides opportunities for students to use their spatial/spatial/spatial abilities to solve problems. (Farisdiyanto and Budiarto, 2014: 78)
Based on this recommendation from NCTM, two keywords must be considered in learning geometry. Namely, spatial ability. This spatial ability, usually termed spatial intelligence, is a person's ability to remember the details of space. As a simple example, if a student is asked to explain about a classroom. His ability to space will make him explain the shape of the room, where objects we love, such as doors, windows, and chairs, to the order in which objects are placed in the classroom.
In particular, the subject matter that is an important concern for researchers, in this case, is spatial structure. Students have different abilities in interpreting spatial structure based on the reality of the teaching and learning process.
Therefore, the researcher intends to study more deeply about the geometric thinking stage using one theory, namely the Van Hiele Theory. Prabowo (2011: 76) suggests that several studies have been conducted to prove that learning at the Van Hiele learning stage can help plan learning and provide

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satisfactory results. He stated this based on the five stages of geometric thinking offered by this theory which is very in line with the method of developing students' spatial abilities.
With the spatial ability test, researchers will find Van Hiele's geometric thinking stages at each subject level (high, average, and low).

Van Hiele's Theory
The Van Hiele theory developed by Pierre Marie Van Hiele and Dina Van Hiele Geldof around the 1950s has strongly influenced the learning of school geometry. Several studies that have been conducted prove that learning that emphasizes the Van Hiele learning stage can help plan learning and provide satisfactory results. "Burger and Shaughnessy (1986) reported that students showed consistent behavior in geometric thinking levels according to Van Hiele's thinking levels. Susiswo (1989) concluded that learning geometry using the Van Hiele model is more effective than conventional learning.
According to Van Hiele's theory, a person will go through five stages of thinking development in learning geometry as follows:
Level 0: Visualization Level (Recognition)
This level is also known as the basic stage, the recognition stage, the holistic stage, and the visual stage, and it is also called the recognition level. At this level, new students recognize a shape's name and its overall shape. For example, squares and rectangles look different. "The student can learn names of figures and recognizes a shape as a whole. The example is squares and rectangles seem to be different" (Hoffer, 1979, 1981 in Zalman Usiskin, 1982).
Level 1: Analysis Level
This level is often called the descriptive level. At this level, students can mention the properties of a shape. For example, a rectangle has four right angles. “The student can identify the properties of figures. The example is rectangles have four right angles” (Hoffer, 1979, 1981 in Zalman Usiskin, 1982).
Level 2: Abstraction Level (Order)
This level is also known as the ordering level or the relational level. At this level, students can arrange a logical thought and understand the relationship between one feature and another in shape but cannot operate it in a mathematical system. For example, students can understand simple conclusions but do not understand the proof. "The student can logically order figures and relationships but does not operate within a mathematical system" (Hoffer, 1979, 1981 in Zalman Usiskin, 1982:12).
Level 3: Formal Deduction Level (Deduction)
Students already understand the role of notions, definitions, axioms, and theorems in geometry at this level. At this level, students have begun to be able to compile evidence formally. "The student understands the significance of deduction and the roles of postulates, theorems, and proof. Proofs can be written with understanding" (Hoffer, 1979, 1981 in Zalman Usiskin, 1982:12).
Level 4: Rigor Level
This level is also known as the metamathematical level. At this level, students can reason formally about mathematical systems (including geometric systems) without the need for concrete models as a reference. At this level, students understand that having more than one geometry is possible.
Spatial Abilities
Spatial ability is the ability to solve problems related to the use of three-dimensional space (Sukardi, 2003). Spatial ability is actually nothing more than learning the language of mathematics, arithmetic, and algebra (Maier, 2003). Spatial ability is the main goal in learning mathematics, especially in school geometry learning. Maier (1998) states that: Spatial ability is a human qualification that is relevant to a high degree to our lives. Several studies (Ethington & Wolfe 1984; Gallagher 1989, Tartre 1990) and a meta-analysis (Kleime 1986) show that spatial skills can be used in specific ways for many mathematical tasks in school. Obviously, spatial abilities are used in a wider range than just for solving geometrical exercises. Even in some other subjects e.g, Chemistry (Pribyl & Bodner 1987), Biology (Lord 1990), and Physical Education (Meeker 1991), success is based fundamentally on

spatial abilities. As well as in school, we also profit from a well-developed spatial ability in professional life (Stumpf & Fay 1983).

Elements of Spatial Ability

Maier (1998) states that there are five elements of spatial ability based on several intelligence theories, meta-analyses, and a number of spatial ability studies. The five elements are as follows.

1) Spatial Perception

Spatial perception should not be mixed with the initial stage of acquiring spatial vision.

2) Visualization

Visualization is the ability to visualize a shape that you want to manipulate.

3) Mental Rotation

Mental Rotation is the ability to quickly and precisely rotate a 2 or 3-dimensional shape.

4) Spatial Relations

Spatial Relations means understanding the spatial configuration of an object or part of an object and its relationship to one another.

5) Spatial Orientation

Spatial Orientation is the ability to adjust physically and mentally in space.

METHOD

The type of research used is a case study. Determination of research subjects based on test results and recommendations from the teacher based on the student's ability to communicate and express opinions/thoughts both orally and in writing.

In addition to the researchers themselves as the main instrument, supporting instruments are also used to give spatial ability tests to determine the level of students' spatial abilities. To support the data on the student's mathematical problem-solving process that has been obtained.

To obtain valid data, triangulation of sources is carried out. Triangulation of sources allows researchers to check data's consistency, depth, and accuracy. Testing the credibility of the data by triangulating sources by comparing the results of interviews by both subjects at each level.

The results of the interview transcripts and the results of the spatial ability test were analyzed using content analysis techniques. The steps are as follows: 1) Examining all available data from various sources, namely the results of interviews and tests of the problem-solving process; 2) Perform data reduction; 3) Data Presentation (data display); 4) Making coding that aims to facilitate data categorization and data presentation about the problem-solving process in terms of the prerequisite abilities and cognitive styles of students; 5) The next step is to validate the data; 6) Next, display the data; 7) Develop a category matrix, look for meaning units, condensed meaning units, and create themes (data interpretation). The condensed meaning unit further clarifies or sharpens the meaning of the data obtained; 8) Interpreting data or drawing conclusions.

RESULT AND DISCUSSION

1. Van Hiele Geometric Thinking Level of High Spatial Ability student

In question 1/element of spatial perception, the two subjects of this level could choose the correct answer. When confirmed about the reasons for choosing the answer during the written test and interview, he gave a logical reason that the most appropriate answer was the object that showed the flattest and straight water surface. Furthermore, in this element of spatial perception, the stages of Van Hiele's geometric thinking are found that the subject has. In the visual stage, the subject managed to identify the shape of the space asked in this question, namely the tube, with the perception that the image has a circle and a height that characterizes the tube. In the analysis stage, the subject only mentions two characteristics: (1) consisting of two circles and (2) having a rectangular blanket. And define based on these two characteristics by outlining that the building has a base and a roof and also has a length in the abstraction stage.

In the second question/visualization element, the first subject chose the correct answer, but he did not provide sufficient explanation to support the answer he chose. He was limited to explaining that the triangle's size on the base's side is the same as the other triangles. Meanwhile, the second subject argues that the nets (a) have each side bent upwards enough to form a regular triangular pyramid on the object. Furthermore, the subject's stages of Van Hiele's geometric thinking are found in this spatial visualization element. The subject visually identified that the pyramid in the figure is a triangular pyramid by matching the nets. In the analysis stage, the subject mentions two characteristics, namely: (1) consists of four sides in the form of a triangle, (2) has a vertex and to calculate the base, the formula (base x height) is divided by 2. It's just that in the abstraction stage, all the properties that the subject mentioned is not used to describe the definition of this triangular pyramid. The subject only mentions that a triangular pyramid is a geometric figure with triangular sides.
In the third question, both subjects chose the correct answer (D) and provided an accurate and logical explanation by describing the difference in form between the correct answer he chose and the other answer choices. In addition, based on his observations, he sees the direction of rotation of the object and estimates the rotation in degrees. Furthermore, the subject's stage of Van Hiele's geometric thinking is found in this mental rotation element. The subject visually identified the rotated object as a cubical space structure on the grounds that it was composed of squares. Then in the analysis stage, the subject mentioned three properties of the cube, namely: (1) consisting of six sides, (2) having 12 edges, and (3) having eight corner points. Again, at the abstraction stage, the subject does not take advantage of all the properties he mentions to be described in definitions. He defined a cube as a shape with six square sides joined together; this problem is a cube. The subject answered the name of the shape correctly, but he was the subject who mentioned the least characteristic of this spatial structure, which was only 1 feature.
In the fourth question/spatial relation, the first subject chooses the correct answer and explanation. It determines the correct answer (C) on the grounds that if c is flipped from top to bottom, it is likely to give the exact same image as the object column. While the second subject chooses the correct answer (C) by comparing objects through the sides that he thinks, have the same four black areas. However, this argument is considered less clear and does not sufficiently support the correctness of
his chosen answer.

The subject chooses the correct answer in the fifth question/spatial orientation. He believes the woman's face is facing oblique to the left and the shape of her hand is most similar to the object in question.
The discussion above is relevant to the findings of JK Alex and JK Mammen (2016: 184), who interviewed a student named Nana. From that research, they found that no matter how simple the student's answer was, they could go through the stages of visualization, analysis, and abstraction in sequence.

2. Van Hiele Geometry Thinking Stage of Average Spatial Ability student

In the first question/element of spatial perception, the first subject was less observant in observing the questions so he chose the wrong answer. His fault is that he only focuses on the appropriate diameter of the line (water), so he thinks answer (C) is the most correct. He forgot that the water's surface must always be flat, in this case, the line on the object. Meanwhile, the second subject chose the correct answer (B) because only the B part of the surface was straight, almost like the angle or diagonal. According to him, if the picture on the problem is in normal condition, only B has a flat surface that matches the picture on the object column.
Furthermore, in this element of spatial perception, the geometric thinking stages of the subject are found. In the visual stage, the subject identifies the shape of the space in question as a tube; he argues that one of the sides is a circle so that it has a diameter. Then, the subject mentioned four properties of the tube in the analysis stage. Among them, (1) have a circular base, (2) a rectangular tube blanket, (3) have a diameter, and (4) have a radius. In the abstraction stage, the subject elaborates on the definition

by mentioning some of these properties. Namely, the tube is a cylindrical shape where the base and roof are circular, and the blanket is rectangular.

In the second question/spatial visualization, the first subject gave the correct answer (A). He got the answer by trying to make the same image and unite the sides of the outer triangle to form a pyramid as shown in the image in the object column. Meanwhile, the second subject explained that first, he observed the design of the base of the image on the object column. He found that (A) was the most appropriate because all left was to unite the three outer triangles to form a central point (the vertex). Furthermore, the subject's stages of Van Hiele's geometric thinking are found in this spatial visualization element. The subject visually identified that the pyramid shape is a regular triangular pyramid by observing its side, an equilateral triangle. Then, the subject mentioned four properties of a triangular pyramid in the analysis stage. Among them, (1) the base is triangular, (2) it has four sides, (3) it has six edges, and (4) it has four vertices. Only some of these properties are used by the subject to elaborate the definition at the abstraction stage. He argued that a triangular pyramid is a geometric figure composed of four equilateral triangles, where the base is triangular.

In the third question, the subject chooses the correct answer (D) only by counting the number of cubes that make up the image on the object. Then the image on the object is simply tilted to the right a little. Next, the shape of the space asked in this question is a cube. The subject answered the name of the space correctly, then mentioned 3 of its characteristics. Furthermore, the subject's stage of Van Hiele's geometric thinking is found in this mental rotation element. The subject visually identified the rotated object as a cube space. Then he analyzed that the subject had four characteristics, namely: (1) composed of a rectangular shape, (2) had six sides, (3) had eight vertices and (4) had 12 edges. From these properties, the subject moves to the abstraction stage by explaining that a cube's definition is composed of six squares with six sides, eight corner points, and 12 edges.

In the fourth question/spatial relation, the first subject chooses the correct answer (C) by looking for an image opposite the image in the object column. According to him, if the image on the right (rhombus) is changed in its position in front of changing positions with a circle, the image below may be a square which then changes its position upwards to replace the cross image. Meanwhile, the second subject failed by choosing the wrong answer, namely (B). His mistake was that he did not look for other possible answers that were closer to the object. It only focuses on B and thinks the circle is only rotated to the right, even though if the circle is rotated to the right, the image above on the object should still be crossed.

In the fifth question/spatial orientation, the first subject was wrong in choosing the answer (B). By mistake, he focused on picture (B) which shows the back side of the picture. According to him, if we raise our right hand automatically on the rear camera, what will be displayed later or taken automatically must be the image behind us and with our right hand. Meanwhile, the opposite happened to the second subject. He chose the correct answer, namely (D), with the argument that D is the most suitable for objects with the same point of view. In addition, he matched objects and answers with the identification of the woman's hand and face on the question.

This moderately capable subject is a subject that can be said to have a high level of geometric thinking because he is the one who mostly mentions the properties of all spatial figures in the given spatial ability test. This is relevant to the findings of Hasan Unal et al. (2009: 1008), who gave a test to a student named Barbara, they did find that Barbara was at a moderate level of ability, but she was much better at distinguishing the types of triangles and mentioned most of the properties of triangles. A distinguishing trait from students at high ability levels is Allen.

3. Van Hiele Geometry Thinking Stage of Low Spatial Ability Subjects

In the first question/spatial perception, the first subject chose the wrong answer (C). Although the subject understood that the space in the glass was divided in two, he failed to analyze that the water surface in the correct image should be a flat surface. Meanwhile, the second subject also chose another wrong answer (A). In addition, the subject cannot account for his choice by providing a strong reason. Furthermore, in this element of spatial perception, the geometric thinking stages of the subject are found. In the visual stage, the subject identified that the shape of the space in the object column was a tube. He assumed that the object was a tube because the base of the glass was circular. In the analysis stage, the subject mentions three characteristics of this spatial structure, namely: (1) having a blanket, (2) the base and roof are circular, and (3) having a center line. In contrast to other subjects, in the abstraction stage, he added an explanation related to volume in the definition of the tube. He argues that the tube is a shape that is influenced by two circular caps in calculating the volume.

In the second question/spatial visualization, the two subjects chose the correct answer (A), assuming that because the pyramid's tip is always pointed (pyramid), they just need to combine the three outer triangles. Furthermore, the subject's stages of Van Hiele's geometric thinking are found in this spatial visualization element. The subject visually identified that the pyramid in the picture is a triangular pyramid, assuming that the shape has a pointed end at each corner. In the analysis stage, the subject mentioned two properties of the pyramid: (1) it is composed of four triangles, (2) it has a height which is the main reference in calculating its volume. Then in the abstraction stage, the subject did not mention any of the properties he had previously written. According to the subject, a triangular pyramid is a spatial shape based on the formulation of interconnected triangles.

In the third question, the first subject chooses the correct answer (D) by looking at the rotation of the object. The subject identified visually that the pyramid in the picture was a cube without explaining the reason. Then, in the analysis stage, the subject mentioned three properties of the cube, namely: (1) it has six sides, (2) it has eight vertices, and (3) it has 12 edges. In the abstraction stage, the subject describes the definition of a cube based on some of the properties he has mentioned before and adds an explanation related to volume. He argues the cube is a wake consisting of six squares which are the reference point for measuring its volume.

In the fourth question, the first subject chose the wrong answer (A). The mistake is because they think A is a choice with many similarities even though if you pay attention, there are objects that have changed, so A is not the correct answer. While the second subject also gave another wrong answer (B) and could not explain why he chose that answer.

In the fifth question, both subjects chose the wrong answer (A). The error lies in the subject not observing the expression and style of the female image in the object column. He was also unable to explain the strong reason why A was the correct answer, according to him.

CONCLUSION AND SUGGESTION

From the profile described above, it can be concluded that a person's high spatial ability does not guarantee that he or she passes through the geometric thinking stages perfectly.

1) At the high level of spatial ability, it is shown that in the analysis stage, he only mentions two to three properties.
2) At the average level of spatial ability, it is shown that in the analysis stage, he mostly mentions four to five properties; in other words, he has the best geometric thinking stage among other students.
3) At the low level of spatial ability, it is shown that in the analysis stage, he mentions 2 to three, but the difference with the others is that he relates the role of these properties in the volume formulation.

Based on the results of the discussion and conclusions that have been presented, the things that researchers need to suggest are as follows:

1) Mathematics teachers at the secondary education level are expected to recognize students' potential in learning mathematics, especially geometry which has a wide application in everyday life.
2) It is hoped that educators do not see students in mathematics learning in one sense; for example, their thinking stage is low, and it could be that their spatial abilities are high.

Needs to do further research using various measuring instruments because the drawback of this research is that it only uses written tests and interviews as measuring tools.

REFERENCES

Alex, Jogymol K. & Mammen, Kuttickattu J. 2016. Geometrical Sense Making: findings of Analysis Based on the Characteristics of the Van Hiele Theory among a Sample of South African Grade 10 Learners. Eurasia Journal of Mathematics, Science and Technology Education. Vol 12 Pages 177-188

Farisdiyanto, Donny Dwi & Mega Teguh Budiarto. 2014. Profile of the Spatial Ability of Junior High School Students in Solving Geometry Problems in View of Differences in Mathematical Ability. Mathedunesa. Volume 3 No. 2 pages 77-84

Maier, Peter Herbert. 1998. Spatial Geometry and Spatial Ability - How to Make Solid Geometry Solid? Selected Papers from the Annual Conference of Didactics of Mathematics 1996. Elmar Cohors-Fresenborg et all (ed). Osnabrueck, 1998, ISBN 3-925386-40-8, pages 63-75.

NCTM (2000). Principals and Standards for School Mathematics. National Council of Teachers of Mathematics. Reston: VA.

Prabowo, Ardhi & Eni Ristiani. 2011. Design of Spatial Ability Test Instruments Development of Herbert Maier's Spatial Ability Test and Identification of Scoring Based on Van Hielle Theory. Semarang: Department of Mathematics, State University of Semarang. Volume 2 Number 2 p. 72-87

Unal, H., Jakubowski, E. & Corey, Derryl. 2009. Differences in Learning Geomtry among High and Low Spatial Ability Pre-service Mathematics Teachers. International Journal of Mathematics Education in Science and Technology. Vol.40 No.8 Pg 997-1012.

Usiskin, Zalman. 1982. Van Hiele Levels and Achievement in Secondary School Geometry. America: University of Chicago.