

Implementation of the Reyleigh-Ritz Method in Exploring Mode Shape Frequency of Free Vibration on Fixed Free Beam using Python Programming

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Abstrak. Balok bebas tetap adalah salah satu komponen yang paling sering digunakan sebagai bahan baku furnitur dan konstruksi bangunan. Penggunaan balok kayu tidak terlepas dari beberapa hal penting yang harus diperhatikan, antara lain kekakuan, kekuatan, stabilitas, dan kelenturan. Masalah getaran bebas yang terjadi pada balok kayu juga harus diperhatikan. Semua sistem yang memiliki massa dan elastisitas dapat mengalami getaran bebas atau getaran yang terjadi tanpa adanya rangsangan dari luar. Getaran bebas ini terjadi ketika sistem pada benda berosilasi akibat kerja gaya-gaya yang ada pada sistem itu sendiri (inheren). Getaran bebas balok kayu harus diatasi dengan redaman, sehingga tidak terlalu mempengaruhi struktur kekuatan, stabilitas, dan kelenturan balok kayu. Redaman dapat dilakukan dengan mencari titik getar bebas atau bentuk modus getar pada balok bebas tetap. Salah satu pendekatan yang dapat digunakan untuk mencari bentuk modus adalah pendekatan numerik dengan menggunakan metode Reyleigh-Ritz. Metodologi yang digunakan adalah pengembangan script python untuk mencari mode getar bebas pada balok. Pada artikel ini, balok dengan panjang 0.5 meter dan tebal 2.25×10^{-4} meter dipilih sebagai model simulasi.

Kata kunci: Bentuk Mode; Python; Metode Reyleigh-Ritz; Fixed Free Beam

Abstract. Fixed free beam is one of the components that are most often used as raw materials for furniture and building construction. The use of wooden beams is inseparable from several important issues that must be taken into account, including stiffness, strength, stability, and flexibility. The problem of free vibration that occurs in wooden beams must also be taken into account. All systems that have mass and elasticity can experience free vibrations or vibrations that occur without external stimulation. This free vibration occurs when the system on the object oscillates due to the work of the forces that exist in the system itself (inherent). The free vibration of the wooden beams must be overcome by damping, so that it does not seriously affect the structure of the strength, stability, and flexibility of the wooden beams. Damping can be done by looking for free vibration points or vibration mode shapes on fixed free beams. One approach that can be used to find the mode shapes is a numerical approach using the Reyleigh-Ritz method. The methodology used is the development of a python script to find free vibration modes on the beam. In this article, the beam with a length of 0.5 meters and a thickness of 2.25×10^{-4} meters is selected as a simulation model.

Keywords: Mode shapes; Python; Reyleigh-Ritz method; Fixed Free Beam

INTRODUCTION

Vibration analysis is one of the vital tasks in designing structural and mechanical systems. Design engineers are interested in learning about how vibration absorbers affect rotating machines, car suspension systems, and the dynamic behavior of machine tool structures under stimulation (Chopade & Barjibhe, 2013).

The system of equations in nonlinear equations that cannot be solved analytically, for example in the fields of science and engineering, often requires finding the roots (solutions), if the equations are in simple form it is easy to determine the roots, but many equations have the form nonlinear problems that are difficult or even have not been solved, but can be solved by special methods for solving the equation problem. To solve the problem of nonlinear equations there are many methods and algorithms that can be used, but each existing method and algorithm has its own advantages and disadvantages, one of which is the numerical method used to solve problems where analytical calculations cannot be used, but iteratively with Using the help of program calculations, you can find an effective solution. There are many kinds of numerical methods for solving systems of linear equations and systems of nonlinear equations including the Rayleigh-Ritz method (Salih, 2018).

Python is a high-level programming language that is very popular today; it can't be separated from the python language which is considered powerful and close to human language. Python's popularity is also due to the increasing need in the fields of Data Science, Machine Learning, Artificial Intelligence, Face recognition, and other fields. Many large companies such as Facebook, Google, Instagram, Netflix, and other digital companies rely on Python as part of their application programming language. The simple structure of the language makes Python very easy to learn. We don't need to think about complicated syntax to do a program (Rangkuti et al., 2021). In it's development of the python, some research has been carried out using python in solving several problems. Zhu et al (2018) using python to develop applications that take advantage of the unique features of scripting languages while leveraging advanced finite element analysis models and algorithms (Zhu et al., 2018). Castro and Jansen (2020) implements Koiter's asymptotic method based on the state-of-the-art collaborative tool, Python (Castro & Jansen, 2021). In the last, De Palma et al. (2019) built a modular analysis program written in Python devoted to the estimation of autocorrelation times for Monte Carlo simulations by means of the Γ -method algorithm (Palma et al., 2019).

In this study, the formulation of the problem that will be discussed is how to find the free vibration form on a fixed free beam using the Rayleigh-Ritz method to exploring mode shapes. This study aims to discuss the Rayleigh-Ritz method and its implementation in solving the problem of the fixed free beam structure. The methodology used in this study is the development of a numerical iteration-based python script that will function as an analytical tool to find free vibration points on fixed free beam. The benefits that are expected from this research are to increase knowledge and can provide a clear picture to the reader about the material discussed.

METHODS

Model Formulation

Beams that are fixed at one end and free at the other are known as Cantilever beams. The schematic of the fixed free beam structure that vibrates when loaded with dynamic forces is shown in the figure below:

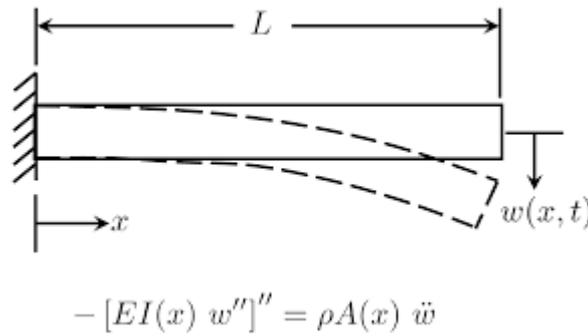


FIGURE 1. The schematic of the fixed free beam structure

From the figure 1, E is known as Young’s Modulus and I is the moment of inertia of the beam, ρ is the mass density, A is cross sectional area of beam, x is distance from placement (m), w is vertical displacement at point $-x$ (m), $L =$ beam length (m). For uniform beam we can obtain equation of motion as (Chopade & Barjibhe, 2013)

$$EI \frac{\partial^4 w}{\partial x^4} - \rho A \omega^2 \frac{\partial^2 w}{\partial x^2} = 0, \tag{1}$$

Here, EI is beam bending stiffness ($N.m^2$), ρA is mass per unit length ($\frac{Kg}{m}$), ω is frequency in radian, t is time. The E, I, ρ and A are constant along the length. Furthermore, Eq (1) can be written as

$$B \frac{\partial^4 w}{\partial x^4} - \frac{\partial^2 w}{\partial x^2} = 0, \tag{2}$$

where $B = \sqrt{\frac{EI}{\rho A}}$ with boundary conditions are

$$w(0) = 0, \quad w(L) = 0. \tag{3}$$

In vibration, mode shapes are the distributions of the relative amplitudes or displacements associated with each mode. Mode represents a pairing of a natural frequency with the corresponding mode shapes. Each natural frequency of a system represents a certain form of vibration system. The order of the vibrations increases from the lowest natural frequency value. A node or node is a point in the distribution of vibration displacements, or in mode shapes. In general, the number of vertices in mode shapes increases with increasing order of vibration (natural frequency).

Rayleigh-Ritz Method for Free Vibration on Fixed Free Beam

Walter Ritz (1878, Switzerland- 1909 Göttingen developed a method which was an extension of the Rayleigh method (Salih, 2018). The Rayleigh-Ritz method has been used for many years to obtain approximations to the solution of partial differential equations and eigenfunction problems. Rayleigh–Ritz method is a direct method for minimizing a given functional. It is direct in the sense that it yields a solution to the variational problem without solving the associated Euler-Lagrange Equation. It may be noted that, for most of the physical problems, the functional we get from the variational principle is not simple and thus the solution using the Euler- Lagrange equation will be difficult to obtain. The Rayleigh–Ritz method is an approximate method where the given functional is directly minimized without recourse to the associated Euler- Lagrange equation. In this method the problem is posed as a minimization problem, say involving an integral. Then some linearly independent functions which satisfy the boundary conditions are chosen. The solution is approximated by a linear combination of these. Finally the coefficients in the approximation are chosen so as to effect minimization. This leads to an eigenvalue problem

involving symmetric matrices. The functions chosen may be, for example, polynomials of low degree, or trigonometric functions. Assume that homogeneous boundary conditions are given. Let $w_k = w_k(x)$ be n functions which satisfy these and approximate the solution w (2) by

$$\tilde{w}(x) = C_1 \sin\left(\frac{\pi}{L} kx\right). \quad (4)$$

In this case, the solution of (4) is obtained. It is the Euler equation associated with minimizing the integral

$$I = \int_{x_1}^{x_2} \left[\frac{p(x)}{2} \left(\frac{dw}{dx}\right)^2 - q(x)w(x) \right] dx. \quad (5)$$

Substituting from Eq. (4) into Eq. (5) we have

$$I = \int_{x_1}^{x_2} \left[\frac{EI}{2} \left(\frac{d^2w}{dx^2}\right)^2 - \rho A \omega^2 w(x) \right] dx.$$

and integration results in a function which may be written

$$I = I(C_1) \quad (6)$$

for functions of the form (4). The minimization of this function and an approximate solution to Eq. (3) is thus obtained by solving

$$\frac{\partial I}{\partial C_1} = 0, \quad k = 1, 2, \dots, n. \quad (7)$$

The effectiveness of the procedure of course depends upon the choice of the approximating functions $w_k(x)$.

Results and Discussions

In this paper, the free vibration of the fix free beam can be denoted as eq. (3) where E is known as Young's Modulus and I is the moment of inertia of the beam, ρ is the mass density, A is cross sectional area of beam, x is distance from placement (m), w is vertical displacement at point - x (m), L = beam length (m). Here, the dimensions and the material constant for a uniform fixed free beam (cantilever beam) studied in this paper are: material of beam = AL , Total length (L) = 0.5 m , width (B) = 0.045 m, height (H) = 0.005 m, moment of inertia (I) = $4.6875 \times 10^{-10} m^4$, Youngs Modulus (E) = 70×10^9 , mass per unit length (m) = 0.30375 kg, mass density (ρ) = 2700 kg/ m³ [1], frequency (ω) = 105.8. Let a trial function

$$y = C_1 \sin\left(\frac{\pi}{L} kx\right)$$

It is the Euler equation associated with minimizing the integral

$$I = \int_0^L \left[\frac{EI}{2} \left(\frac{d^2w}{dx^2}\right)^2 - \rho A \omega^2 w(x) \right] dx.$$

Substituting from Eq. (4) into Eq. (5) we have

$$I = \int_0^L \left[\frac{EI}{2} \left(-9.8696 \left[C_1 \sin \left(\frac{\pi}{L} x \right) \right] \right)^2 - \rho A \omega^2 \left[C_1 \sin \left(\frac{\pi}{L} x \right) \right] \right] dx. \quad (7)$$

and integration results in a function which may be written

$$I = I(C_1) \quad (8)$$

for functions of the form (8). The minimization of this function and an approximate solution to Eq. (3) is thus obtained by solving

$$\frac{\partial I}{\partial C_1} = 0, \quad k = 1, 2, \dots, n. \quad (9)$$

Next, by python, the parameter is obtained as $C_1 = 0.169032298047017$. The trial function can be written as $Y = 0.169032298047017 \sin(\pi * x/0.5)$ for $k = 1$. For various of k values, the solution can be solve through python script. The author has designed a Python script based on jupyter by implementing the Reyleigh-Ritz method as follows:

```
# Import Library
import numpy as np
from scipy import integrate
from matplotlib import pyplot as plt
import sympy
import matplotlib as mpl
#Define parameter
x, k, A, L, E, I, rho, Y0, pi, C, f = sympy.symbols("x, k, A, L, E, I, rho, Y_0, pi, C, f")
Y = sympy.Function("Y")
A=1*0.045*0.005
E= 7*10000000000
I= 4.6875/10000000000
rho= 2700
A = 0.5*0.045*0.005
L =0.5
pi=22/7
omega=105.80
# Rayleigh-ritz Method
f=C*sympy.sin(pi*x/L) # trial function
dphi=sympy.diff(f,x,x) #
IIn=sympy.integrate(E*I*(dphi**2)/2-
omega**2*A*rho*f, (x, 0, L))
```

```

IIn
koef=sympy.diff(IIn, C)
CC=sympy.solve(koef)
Y=CC*sympy.sin(pi*k*x/L)
CC=0.169032298047017
Y=0.169032298047017sin(pik*x/L)
#Plotting
import numpy as np
import matplotlib.pyplot as plt
x = np.linspace(0, 1, 100)
#Plot in various value of k (mode shapes)
line1, = plt.plot(x, CC*np.sin(1*pi*x/L), label='1st mode')
line2, = plt.plot(x, CC*np.sin(2*pi*x/L), label='2nd Mode')
line3, = plt.plot(x, CC*np.sin(3*pi*x/L), label='3rd Mode')
line4, = plt.plot(x, CC*np.sin(4*pi*x/L), label='4th Mode')
line5, = plt.plot(x, CC*np.sin(5*pi*x/L), label='5th Mode')
#legend for mode shape
plt.legend(handles=[line1, line2,line3, line4, line5], loc='lower left')
# Add x-axis label
plt.xlabel('x')
# Add Y-axis label
plt.ylabel('Solution')
#major grid lines
plt.grid(b=True, which='major', color='gray', alpha=0.6, line
style='dashdot', lw=1.5)
#minor grid lines
plt.minorticks_on()
plt.grid(b=True, which='minor', color='beige', alpha=0.8, ls=
'-', lw=1)
plt.show()

```

The user is asked to import some modules for some commands in the form: (1) `import numpy as np`, is a Python library that functions for numerical computing processes, (2) `from scipy import integrate` The use of `scipy` is to handle algebraic and matrix operations as well as other mathematical operations (3) `from matplotlib import pyplot as plt`. (4) `import sympy`, `sympy` is an open source Python library used for symbolic math such as integral operations (5)

`import matplotlib as mpl`. Matplotlib is one of the libraries in Python that is used for the data visualization process. Then the program will continue with defining parameters by writing `x, k, A, L, E, I, rho, Y0, pi, C, f = sympy.symbols("x, k, A, L, E, I, rho, Y_0, pi, C, f")`
`Y = sympy.Function("Y")`

Then input the parameter values. Before using the Rayleigh-ritz method, it is assumed that a trial function $f=C*\text{sympy.sin}(\pi*x/L)$. Then the program will continue the analysis using the Rayleigh-Ritz method to find a solution to the problem of fixed free beam as

```
dphi=sympy.diff(f, x, x) #
IIn=sympy.integrate(E*I*(dphi**2)/2-
omega**2*A*rho*f, (x, 0, L))
IIn
koef=sympy.diff(IIn, C)
CC=sympy.solve(koef)
CC
Y=CC*sympy.sin(pi*k*x/L)
```

Python scripts can be run by pressing the button runtime by choosing run allor Ctrl+F9. After all solutions are found, the program will display a solution graph with various value of shape modes. The users can see where the solution is and how many points through the graph through the following syntax:

```
import numpy as np
import matplotlib.pyplot as plt
x = np.linspace(0, 1, 100)
#Plot in various value of k (mode shapes)
line1, = plt.plot(x, CC*np.sin(1*pi*x/L), label='1st mode')
line2, = plt.plot(x, CC*np.sin(2*pi*x/L), label='2nd Mode')
line3, = plt.plot(x, CC*np.sin(3*pi*x/L), label='3rd Mode')
line4, = plt.plot(x, CC*np.sin(4*pi*x/L), label='4th Mode')
line5, = plt.plot(x, CC*np.sin(5*pi*x/L), label='5th Mode')
#legend for mode shape
plt.legend(handles=[line1, line2, line3, line4, line5], loc='lower left')
# Add x-axis label
plt.xlabel('x')
# Add Y-axis label
plt.ylabel('Solution')
#major grid lines
plt.grid(b=True, which='major', color='gray', alpha=0.6, line
style='dashdot', lw=1.5)
```

```
#minor grid lines
plt.minorticks_on()
plt.grid(b=True, which='minor', color='beige', alpha=0.8, ls=
'-', lw=1)
plt.show()
```

Next, python will display a graph like in Figure 2.

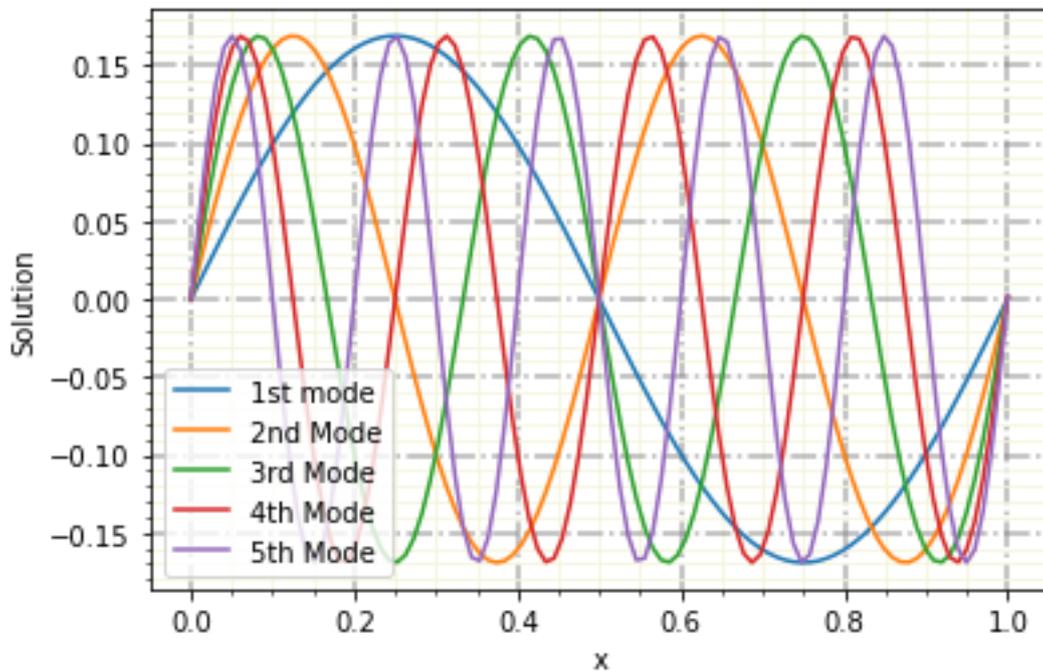


FIGURE 2. Graph of Vibration of Fixed Free Beam for Various Shape Mode Values

CONCLUSIONS

We present a script of program written in Python devoted to obtain the solution of the Rayleigh framework in different mode shapes. We give a brief review of this method and describe the main features of the program. Finally, we present a simple application as an alternative script for the program.

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