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HARMONIOUS LABELING ON HELM GRAPHS

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Abstract
graph $\mathrm{G(V,E)}$ consists of two sets, namely vertices Vand edges E, which Vare sets that cannot be empty. The helmet graph is obtained from graph circle with addition side pendants with notation $\mathrm{H_n}$. Something graph Hside q is said to be harmonious if there is an injective function $\mathrm{f: V \to \{0,1,2,\ldots,3n-1\}}$that produces function labeling side $\mathrm{g(xy) = \big(f(x)+f(y)\big)(mod\ q)}$which will result in a different sided label. In this thesis, graphs $\mathrm{H_n}$with nodd and even nresults will be constructed as $\mathrm{H_n}$ harmonious graphs. Where for every $\mathrm{n \geq 3}$for odd helmet graph and $\mathrm{n \geq 4}$for even helmet graph.

Keywords : Graph Labeling, Helmet graphics, Harmonious Graph , Labeling harmonious.

INTRODUCTION

Labeling graph is mapping one on one map set from point and side graph to set number round positive , or in other words ie pairing function _ every element graph good side nor point with number round positive (Gallian n, 20 20). Labeling graph already many growing , for one is labeling harmonious . Something graph $H$with sides $q$is said to be harmonious if there is an injective function $f: V \to \{0,1,2,\ldots,3n-1\}$ that produces function labeling side $g(xy) = \big(f(x)+f(y)\big)(mod\ q)$ which will result in a different sided label. (Graham and Shloane, 1980).
A number of results research on labeling harmonious among them is P.Jeyanthi (2018) is decisive that super subdivium graph is graph harmonious odd , M. Kalaimathi (2019) is decisive graph acyclic is graph harmonious . Now continues with helmet graph to be showed as graph harmonious .
On research this will given construction labeling harmonious on the helm H_n graph with n odd $\geq 3$and $n$even ≥ 4 which will show that the helm graph is a harmonious graph.

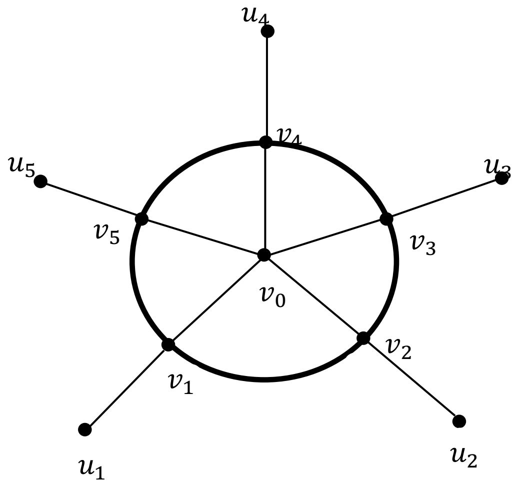
RESULTS AND DISCUSSION

In part this will discussed about results labeling harmoniouss on the helm graph with $n$odd and $n$even. The definition of a set of vertices and a set of edges on a helm graph are $V(H_n) = \{v_0, v_i, u_i \mid i = 1, \ldots, n\}$and $E(H_n) = \{v_0v_i, v_iu_i \mid i = 1, \ldots, n\} \cup \{v_iv_{i+1} \mid i = 1, \ldots, n-1\} \cup \{v_1v_n\}$.

1. ODD HARMONIOUS LABELING

In part this will be discussed about labeling harmony odd with $n \geq 3$ by constructing the point labeling function in the form of an algorithm and then determining the edge labeling.

Figure 1. Helmet graph H_5



Next defined labeling $f: V \rightarrow \{0,1,2,\ldots,3n-1\}$ on helm graph H_n for $n \geq 3$ odd, use the following Algorithm 1:

Algorithm 1:

1. Point center v_0 labeled 0.
2. Point v_1 labeled 1.
3. Point v_i by $i = 2, \ldots, n$ being labeled
$$\frac{3i-2}{2}.$$
4. The pendant point u_1 is labeled
$$3n-1.$$
5. Point pendant u_i with $i = 2,3, \ldots, n$ labeled
$$3(n-i+1).$$

Theorem 1. *Suppose* $n \geq 3$ *and* H_n *is a helmet graph, then it* H_n *is a harmonious graph***.**

Proof:

For prove that helmet H_n graph is a harmonious graph, so a labeling will be constructed on H_n those that fulfill it properties certain . Based on results research conducted on cases _ simple , then , is formed function labeling vertices on the helmet H_n graph with $n$odd numbers $f: V \rightarrow \{0,1,2,\ldots,3n-1\}$ that is :

$$f(v_i) = \begin{cases} 0, & i = 0 \\ 1\,, & i = 1 \\ \left\lceil \dfrac{3i-2}{2} \right\rceil, & i = 2,3,\ldots,n\ , \end{cases}$$

and

$$f(u_i) = \begin{cases} 3n-1, & i = 1 \\ 3(n-i+1), & i = 2,3,\ldots,n\,. \end{cases}$$

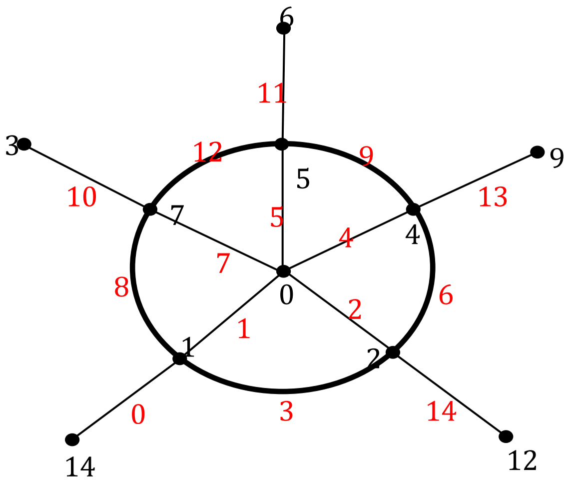
Based on constructed function _ that , next will proven that every side have different weights _ in modulo $3n$by using a function $g(e_i) = f(x) + f(y)$ with $e = xy$, as follows:

1. $g(v_0v_1) = (f(v_0) + f(v_1))\ (mod\ 3n)$
$= 1\ (mod\ 3n).$

2. For $i = 2, \dots, n$, obtained
$$g(v_0v_i) = (f(v_0) + f(v_i))\ (mod\ 3n)$$
$$= \left\lceil \frac{3i-2}{2} \right\rceil (mod\ 3n).$$
3. $g(v_1v_2)\ = (f(v_1) + f(v_2))\ (mod\ 3n)$
$$= 3\ (mod\ 3n).$$
4. For $i = 2, \dots, n-1$, obtained
$$g(v_iv_{i+1}) = (f(v_i) + f(v_{i+1}))\ (mod\ 3n)$$
$$= \left(\left\lceil \frac{3i-2}{2} \right\rceil + \left\lceil \frac{3(i+1)-2}{2} \right\rceil \right) (mod\ 3n)$$
5. $g(v_1v_n)\ = (f(v_1) + f(v_n))\ (mod\ 3n)$
$$= \left(n + \left(\frac{n+1}{2} \right) \right)\ (mod\ 3n).$$
6. $g(v_1u_1)\ = (f(v_1) + f(u_1))\ (mod\ 3n)$
$$= 0.$$
7. For $i = 2, \dots, n$, obtained
$$g(v_iu_i)\ = (f(v_i) + f(u_i))\ (mod\ 3n)$$
$$= \left(\left\lceil \frac{3i-2}{2} \right\rceil + 3(n-i+1) \right) (mod\ 3n).$$

Based on equation (1) to equation (7) is obtained weight different side , with _ thereby function labeling $f^*: E(H_n) \to \{0,1, \dots, 3n-1\}$ defines that a graph H_n with n an odd number is a harmonious graph.

Figure 2. Labeling helmet H_5 graph



2. EVEN HARMONIOUS LABELING

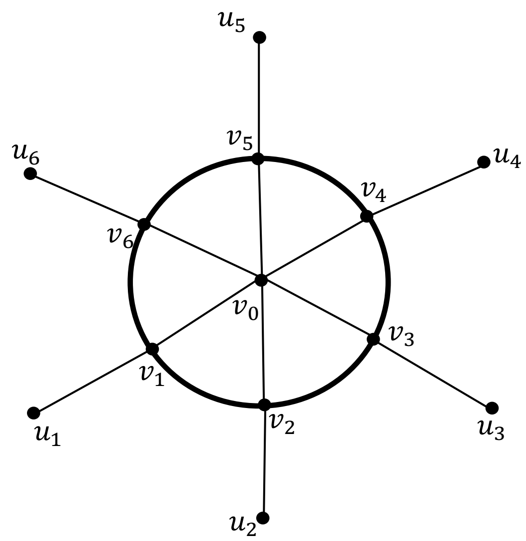
In sub chapters this will discussed about labeling harmonious even in the helm graph . Previously written down return definition set point and set edge on the helmet graph for $n \geq 4$ as follows:

$V(H_n) = \{v_0, v_i, u_i \mid i = 1, \dots, n\}$

and

$E(H_n) = \{v_0v_i, v_iu_i | i = 1, \dots, n\} \cup \{v_iv_{i+1} | i = 1, \dots, n\} \cup \{v_1v_n\}.$

Figure 3. Helmet Graph H_6



Next defined labeling $f: V \rightarrow \{0,1,2,\dots,3n-1\}$ on a helmet graph $H_n$for an $n \geq 4$ even number, the following Algorithm 2 is used:

Algorithm 2:

1. Point main v_0 labeled 0.
2. Point v_1 labeled 1.
3. Point $v_i$by $(i = 2,\dots,n-1)$ being labeled
$$\frac{3i-2}{2}.$$
4. Point v_n labeled
$$\frac{3n+2}{2}.$$
5. The pendant point u_1 is labeled
$$3n-1.$$
6. The pendant point u_2 is labeled
$$3n-4.$$
7. The pendant point u_3 is labeled
$$3n-7.$$
8. Point pendant u_i with $i = 4,\dots,n-1$ labeled
$$3(n-i-1).$$
9. The pendant point u_n is labeled
$$3n-2.$$

Based on results research , obtained Theorem 2 follows :

Theorem 2. *Suppose $n \geq 4$ and H_n is a helmet graph, then it H_n is a harmonious graph .*

Proof:

For prove that helm H_n graph is a harmonious graph, so a label will be constructed on those that meet H_n certain properties . Based on results research conducted on cases _ simple , then , is formed function labeling vertices on the helmet H_n graph with $n$an even number $f: V \rightarrow \{0,1,2,\ldots,3n-1\}$

$$f(v_i) = \begin{cases} 0, & i = 0 \\ 3i-2, & i = 1 \\ \left\lceil \dfrac{3i-2}{2} \right\rceil, & i = 2,3,\ldots,n-1 \\ \left\lceil \dfrac{3n+2}{2} \right\rceil, & i = n \end{cases}$$

and

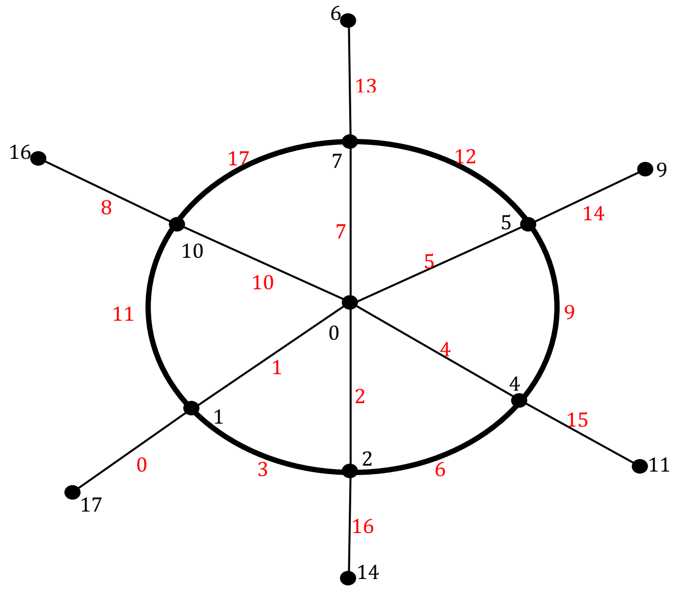
$$f(u_i) = \begin{cases} 3n-1, & i = 1 \\ 3n-4, & i = 2 \\ 3n-7, & i = 3 \\ 3(n-i-1), & i = 4,\ldots,n-1 \\ 3n-2, & i = n\,. \end{cases}$$

Based on constructed function _ that , next will proven that every side have different weights _ in modulo $3n$ by using a function $g(e) = f(x) + f(y)$ with $e = xy$, as follows:

1. $g(v_0v_1) = (f(v_0) + f(v_1))\ (mod\ 3n)$
   $= 1\ (mod\ 3n).$
2. For $i = 2,\ldots,n-1$
   $g(v_0v_i) = (f(v_0) + f(v_i))\ (mod\ 3n)$
   $= \left\lceil \frac{3i-2}{2} \right\rceil (mod\ 3n).$
3. $g(v_0v_n) = (f(v_0) + f(v_n))\ (mod\ 3n)$
   $= \left\lceil \frac{3n+2}{2} \right\rceil (mod\ 3n).$
4. For $i = 1$
   $g(v_iv_{i+1}) = (f(v_1) + f(v_2))\ (mod\ 3n)$
   $= 3\ (mod\ 3n).$
5. For $i = 2,\ldots,n-1$
   $g(v_iv_{i+1}) = (f(v_i) + f(v_{i+1}))\ (mod\ 3n)$
   $= \left(\left\lceil \frac{3i-2}{2} \right\rceil + \left\lceil \frac{3(i+1)-2}{2} \right\rceil\right) (mod\ 3n).$
6. $g(v_1v_n) = (f(v_1) + f(v_n))\ (mod\ 3n)$
   $= \left(1 + \left\lceil \frac{3n+2}{2} \right\rceil\right) (mod\ 3n).$
7. $g(v_1u_1) = (f(v_i) + f(u_i))\ (mod\ 3n)$
   $= 0.$
8. $g(v_2u_2) = (f(v_2) + f(u_2))\ (mod\ 3n)$
   $= (3n-2)\ (mod\ 3n).$
9. $g(v_3u_3) = (f(v_3) + f(u_3))\ (mod\ 3n)$
   $= (3n-3)\ (mod\ 3n).$
10. For $i = 4,\ldots,n-1$
    $g(v_iu_i) = (f(v_i) + f(u_i))\ (mod\ 3n)$
    $= \left(\left\lceil \frac{3i-2}{2} \right\rceil + 3(n-i+1)\right) (mod\ 3n).$
11. $g(v_nu_n) = (f(v_n) + f(u_n))\ (mod\ 3n)$
    $= \left(\left\lceil \frac{3n+2}{2} \right\rceil + (3n-2)\right) (mod\ 3n).$

Based on equality on show that every side have different weights , with _ thereby function labeling $f^*: E(H_n) \rightarrow \{0,1,\ldots,3n-1\}$ defines that a graph $H_n$with n an even number is a harmonious graph.

Figure 4. Labeling helmet H_6 graph



CONCLUSION

Based on results study so obtained conclusion that helmet H_n graph which is shown in Theorem 1 and Theorem 2 is something graph harmonious .

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