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AN EMPIRICAL STUDY FOR COMPARISON OF ESTIMATION METHODS FOR VALUE AT RISK, TAIL VALUE AT RISK, AND ADJUSTED TAIL VALUE AT RISK USING EXTREME VALUE THEORY APPROACH TO STOCK MARKET INDEX

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Abstract
Risk management helps the financial industry to manage and estimate the risks that may occur by using risk measures. Financial series data mostly have a heavy tail distribution which causes the probability of extreme values to occur. To overcome these extreme values, it is necessary to apply a mathematical model in calculating risk estimates in financial data combined with the Extreme Value Theory approach. The Adjusted-TVaR model is a measure of the risk of modification of the TVaR model to eliminate outliers in the tail of the distribution. The purpose of this study is to measure the accuracy of the Value at Risk, Tail Value at Risk, and Adjusted Tail Value at Risk using the Peak Over Threshold approach in Extreme Value Theory Models. The results of the risk estimation research using the POT approach method, show that the higher the level of confidence and the chosen constant, the higher the value of Adj-TVaR presented. This value represents that the potential loss will be higher. The estimation results obtained that the VaR value is smaller than Adj-TVaR and Adj-TVaR is smaller than TVaR. This shows that Adj-TVaR is more efficient to use in terms of predicting risk value when compared to TVaR with the Peak Over Threshold approach

Keywords: Risk, Value at Risk, Tail Value at Risk, Adjusted TVaR, Extreme Value Theory, Peak Over Threshold.

INTRODUCTION

Risk management helps the financial industry to manage and estimate the risks that may occur by using risk measures. The risk measure commonly used is Value at Risk (VaR). According to Jorion [1] VaR is defined as the maximum loss that can occur within a contract period with a certain level of confidence. VaR gives a vision of the risks that can occur. In practice, the loss value is usually higher than the estimated VaR value. Carmona [2] shows that VaR as a risk measurement tool has two weaknesses, namely VaR does not calculate the actual size of losses, and VaR does not allow diversification. In addition, VaR also does not meet the nature of coherence [3] even though it is important in risk management. To overcome the weaknesses of VaR, several researchers have introduced another risk measure that is coherent, namely Tail Value at Risk (TVaR).

The TVaR method can be interpreted as the average value of the loss that will be borne, in the event of a loss that exceeds the VaR. The TVaR method is coherent. However, the main weakness of the TVaR model is that it is very dependent on the random risk distribution whose value is more than the VaR estimate, so that if there is an outlier value or extreme loss value that is greater than VaR, the estimation results from TVaR will be inaccurate. Based on these problems, the Adjusted-TVaR (Adj-TVaR) model is introduced as a modification of the TVaR model. Research conducted by Trimono [4] examines the upper limit of the aggregate risk measure by using Adj-TVaR as a modified form of TVaR to eliminate outliers in the tail of the distribution.

The Indonesia Stock Exchange (IDX) stated that the stock price index experienced a decline in March 2020, due to the Covid-19 pandemic. Many companies/investors sell their shares because of the volatility which causes greater risk and uncertainty faced by investors. Dayong Zhang, et al [5] showed that the risk of global financial markets has increased as a result of the Covid-19 pandemic. With the uncertainty surrounding the Covid-19 pandemic and the economic losses that have caused financial markets to become so volatile and unpredictable that it can cause long-term issues, there is a clear correlation between the stock market response and the severity of the outbreak in each country.

Financial series data mostly have a heavy tail distribution which causes the probability of extreme values to occur. Extreme events that are rare but have a big impact, so to overcome these extreme values, it is necessary to apply a mathematical model in calculating risk estimates in financial data combined with the Extreme Value Theory (EVT) approach [6]

The application of EVT in estimating risk measures so far has been limited to the VaR, ES, and ARMA GARCH methods. No research has been conducted on the EVT application on Adjusted-TVaR. Based on this description, in this study the authors took the initiative to conduct research using the Adjusted-TVaR (Adj-TVaR) risk measure which can overcome the outlier values that occur by using the Extreme Value Theory (EVT) approach which is related to the extreme values that appear in the data financial crisis or during the Covid-19 pandemic.

METHODS

Extreme Value Theory (EVT) is a statistical method regarding the deviation of data from the average value in the probability distribution that focuses on tail behavior that model events that contain extreme values. To determine the probability of extreme values with heavy-tail data which cannot be done with the usual approach where the events modeled are extreme which rarely occur but have a large enough impact and cannot be done with a normal distribution because the financial data are not normally distributed. The EVT method can be identified by two approaches, which the Block Maxima (BM) method, which takes the maximum value in one period and the Peak Over Threshold (POT) method, which takes values that pass a threshold value [7].

The Peak Over Threshold (POT) method is one of the methods to calculate the distribution of extreme data exceeding the specified threshold value called the threshold. The extreme value is above the threshold [8]. It is known that the function of a distribution F of the random variable x, and estimated by the distribution function *Fu* from the value of x above the threshold value (u). The theorem was introduced by Pickands (1975), Balkema and de Haan (1974) discussed by Gilli and Kellezi [9] which states that the higher the threshold value (u), the extreme data will follow the Generalized Pareto Distribution (GPD) distribution with cumulative distribution. function (cdf) is as follows:

$$F(x) = \begin{cases} 1 - \left(1 + \frac{\xi(x-u)}{\sigma}\right)^{\frac{-1}{\xi}}, & \begin{matrix} 0 \le x - u < -\frac{\sigma}{\xi} \quad jika \;\; \xi < 0 \\ 0 \le x - u < \infty \quad jika \;\; \xi > 0 \end{matrix} ; \\ 1 - e^{-\left(\frac{x-u}{\sigma}\right)}, \quad 0 \le x - u < \infty, & jika\; \xi = 0. \end{cases} \tag{1}$$

Where ξ is the parameter of the shape (tail index) and σ is the parameter Scale.

The parameter estimation of the GPD method can be estimated using the maximum likelihood method. The steps are as follows:

1. Taking n random samples $x_1, x_2, x_3, \dots, x_n$ by taking values that exceed a predetermined threshold.
2. Establish a probability density function (pdf) for the distribution of GPD

$$f(x) = \begin{cases} \frac{1}{\sigma}\left(1+\frac{\xi(x-\mu)}{\sigma}\right)^{\frac{-1}{\xi}-1}, jika\ \xi \neq 0\ ; \\ \\ \frac{1}{\sigma}\exp\left(-\frac{x-\mu}{\sigma}\right), \qquad jika\ \xi = 0; \end{cases} \tag{2}$$

3. Create a likelihood function where the likelihood function for GPD is as follows:

$$\begin{aligned} L(\mu,\sigma,\xi) &= \prod_{i=1}^{n} f(x_i) \\ &= \prod_{i=1}^{n} \frac{1}{\sigma}\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{\frac{-1}{\xi}-1} \\ &= \left(\frac{1}{\sigma}\right)^{n} \prod_{i=1}^{n}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{-1}{\xi}-1}. \end{aligned} \tag{3}$$

4. Establish the ln likelihood function for the GPD distribution as shown in the following equation:

$$\ln L(\mu,\sigma,\xi) = -n\ln(\sigma) - \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{n}\ln\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right]^{\frac{-1}{\xi}-1}. \tag{4}$$

5. Then the next step is to maximize the ln likelihood function by finding the first derivative of the distribution parameter (μ,σ,ξ). The results of the first derivative of ln likelihood for $\xi \neq 0$.

$$\frac{\partial \ln L(\mu,\sigma,\xi)}{\partial \mu} = \left(\frac{1+\xi}{\sigma}\right)\sum_{i=1}^{n}\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{-1} = 0. \tag{5}$$

$$\frac{\partial \ln L(\mu,\sigma,\xi)}{\partial \sigma} = -\frac{n}{\sigma} + (1+\xi)\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma^2}\right)\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)^{-1} = 0. \tag{6}$$

$$\begin{aligned} \frac{\partial \ln L(\mu,\sigma,\xi)}{\partial \xi} &= \frac{1}{\xi^2}\sum_{i=1}^{n}\ln\left[1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right] \\ &\quad - \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{n}\left(1+\xi\left(\frac{x_i-\mu}{\sigma}\right)\right)\left(\frac{x_i-\mu}{\sigma}\right) = 0. \end{aligned} \tag{7}$$

6. Based on the above equation, it is obtained that the equation is not in close form, so to be able to estimate the GPD parameters, a numerical method will be used.

Value at Risk (VaR) is a type of risk measure that is popularly used, in general VaR is defined as the maximum expected loss of an asset or stock value in a certain period with a certain level of confidence [9]. Definition: Suppose $0 < \alpha < 1$ and F is the distribution function of the random variable X is the loss rate of an investment in a certain period. VaR is the inverse form of the cumulative distribution function (cdf) [11]

The calculation of VaR for the GPD distribution is by obtaining the inverse function of the cumulative distribution function, from the cumulative distribution of GPD in equation (1) So that the VaR value can be obtained using the GPD approach with the following formula:

$$VaR_{\alpha}(X) = F_x^{-1}(\alpha). \tag{8}$$

The calculation of VaR for the GPD distribution is by obtaining the inverse function of the cumulative distribution function, from the cumulative distribution of GPD in equation (1) So that the VaR value can be obtained using the GPD approach with the following formula:

$$VaR_{\alpha(GPD)} = u + \frac{\hat{\sigma}}{\hat{\xi}}\left[\left[\frac{n}{N_u}(1-\alpha)\right]^{-\hat{\xi}} - 1\right]. \tag{9}$$

Where:
u : *Threshold,*
n : Number of observations,
N_u : The number of observations that are above the threshold u,
$\hat{\xi}$: Shape estimation parameter (*shape*)/*tail index,*
$\hat{\sigma}$: Scale estimation parameter (*scale).*

The VaR risk measure is not a coherent risk measure. Therefore, TVaR as an alternative risk measure is better than VaR. TVaR can be interpreted as the average value of the loss that will be borne, in the event of a loss that exceeds VaR.
Definition: Let X be the random variable X which represents the random variable of loss. The Tail Value at Risk of X at the 100α% security level, expressed as $TVaR_\alpha(X)$ is the expected loss value known that the loss exceeds the 100 percentile or quantile of the X distribution.
The formula $TVaR_\alpha(X)$ can be written as follows [12]

$$TVaR_\alpha(X) = E(X|X > \pi_\alpha) = \frac{\int_{\pi_\alpha}^{\infty} xf(x)dx}{1-F(\pi_\alpha)}. \tag{10}$$

In another form $TVaR_\alpha(X)$can be formulated as follows:

$$TVaR_\alpha(X) = E(X|X > \pi_\alpha) = \frac{\int_\alpha^1 VaR_u(X)du}{1-\alpha}. \tag{11}$$

TVaR can be seen that the average of all VaR values is above the 100% security level, which means that TVaR has more tails of distribution than VaR. In another form $TVaR_\alpha(X)$ can be formulated as follows:

$$\begin{aligned} TVaR_\alpha(X) &= E\big(X\big|X > VaR_\alpha(X)\big) \\ &= VaR_\alpha(X) + \frac{1}{1-\alpha}E\big[\big(X - VaR_\alpha(X)\big)\big]. \end{aligned} \tag{12}$$

TVaR with the GPD approach can be obtained by the following equation:

$$\begin{aligned} TVaR_\alpha(X) &= E\big(X\big|X > VaR_\alpha(X)\big) \\ &= E\big(X\big|X + VaR_\alpha(X)\big) \\ &= E[VaR_\alpha(X)|X > VaR_\alpha(X)] + E[X - VaR_\alpha(X)|X > VaR_\alpha(X)] \\ &= VaR_\alpha(X) + E[X - VaR_\alpha(X)|X > VaR_\alpha(X)] \\ &= VaR_\alpha(X) + \frac{\hat{\sigma}+\hat{\xi}(VaR_\alpha(X)-u)}{1-\hat{\xi}} \\ &= \frac{VaR_\alpha(X)}{1-\hat{\xi}} + \frac{\hat{\sigma}+\hat{\xi}\,u}{1-\hat{\xi}}. \end{aligned} \tag{13}$$

Where:
u : *Threshold,*
$VaR_\alpha(X)$: *Value at risk*,
$\hat{\xi}$: Shape estimation parameter (*shape*)/*tail index,*
$\hat{\sigma}$: Scale estimation parameter (*scale).*

Let X be a single random risk. The TVaR model can be viewed with the average value of loss from random risk X which is greater than VaR. Therefore, TVaR is highly dependent on a random risk distribution whose value is greater than VaR. If at the tail of a very high random risk distribution (outliers), the TVaR values obtained are biased and less accurate, this is the main weakness of the TVaR model. So to correct the weakness of TVaR, Jadhav [13] introduced the Adjusted-TVaR (Adj-TVaR) risk size model which is a modification of the TVaR model. The Adj-TVaR model is expected to be able to eliminate the effect of inaccurate TVaR estimation caused by the outlier value in the tail of the distribution. The definition for Adj-TVaR according to Jadhav [13] is as follows:

Definition: For a random loss X and some $\alpha \in (0,1)$ and $c \in [0,0.1]$, $Adj-TVaR_{(a,c)}(X)$ is defined as the mean loss in the interval between $VaR_{\alpha}(X)$ and $VaR_{\alpha+(1-\alpha)^{1+c}}(X)$. Therefore,

$$Adj-TVaR_{(a,c)}(X) = E[X|VaR_{\alpha}(X) \leq VaR_{\alpha+(1-\alpha)^{1+c}}(X)]. \quad (14)$$

In equation (14) let $VaR_{\alpha}(X) = a$, dan $VaR_{\alpha+(1-\alpha)^{1+c}}(X) = b$. We obtain

$$\begin{aligned} Adj-TVaR_{(a,c)}(X) &= \frac{1}{P(a \leq X \leq b)} \int_a^b x f_X(x) dx \\ &= \frac{1}{F_X(b) - F_X(a)} \int_a^b x f_X(x) dx \\ &= \frac{1}{(1-\alpha)^{1+c}} \int_a^b x f_X(x) dx. \end{aligned} \quad (15)$$

By substituting $F_X(x) = \mu, x = F_X{}^{-1}(\mu), and\ f(x)dx = d\mu$,

$$Adj-TVaR_{(a,c)}(X) = \frac{1}{(1-\alpha)^{1+c}} \int_{\alpha}^{\alpha+(1-\alpha)^{1+c}} F_X{}^{-1}(\mu) d\mu. \quad (16)$$

Adj-TVaR with the GPD approach can be obtained by the following equation:

$$Adj-TVaR_{(a,c)(GPD)}(X) = u + \frac{\hat{\sigma}}{\hat{\xi}} \left(\left(\frac{n}{N_u}\right)^{-\hat{\xi}} \frac{\left((1-\alpha)^{1+c}\right)^{-\hat{\xi}}}{-\hat{\xi}+1} - 1 \right). \quad (17)$$

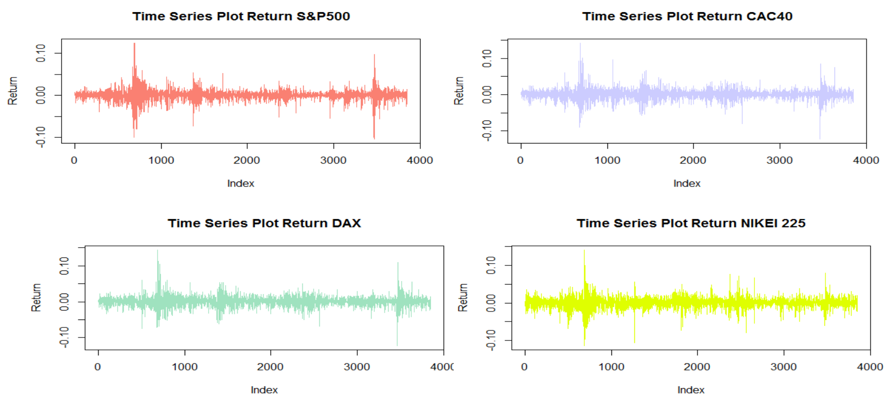
Where:

u : *Threshold*
n : The number of returns,
Nu : The number of extreme data,
$\hat{\xi}$: Shape estimation parameter,
$\hat{\sigma}$: Scale estimation parameter.

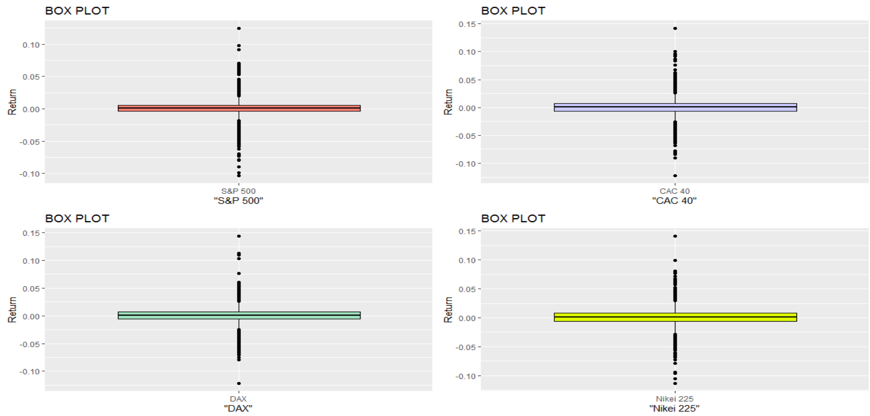
.

RESULT AND DISCUSSION

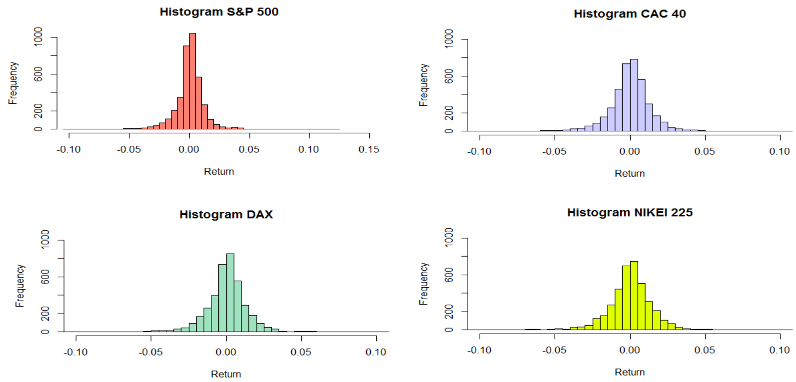
The data used in this study is data on several daily closing prices (closing prices) of international stock market indices in the world which include S&P 500 (US; SPX), DAX (Germany; GDAXI), CAC 40 (France; FCHI) and Nikkei 225 (Japan; N225) in the period January 1, 2006 to September 30, 2021. To see the pattern of movement of stock index returns, see the time series plot in Figure 1:



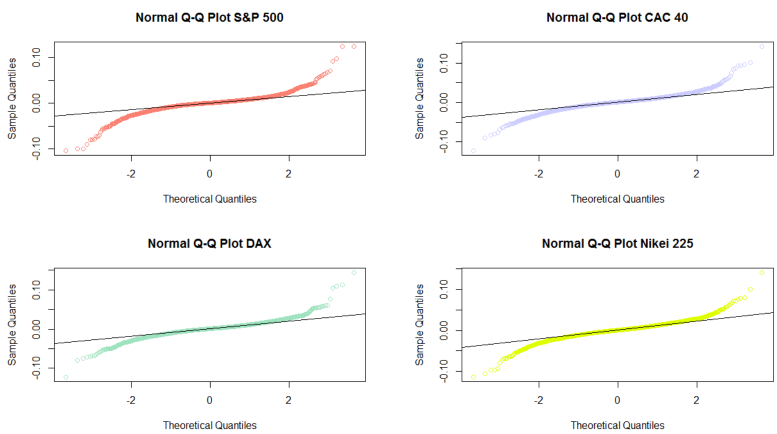
From the data anlysis there is a skewness value, there is a kurtosis value which shows the distribution of the data or a value that measures the tail of the data. The kurtosis value in Table 1 for each stock market return has a kurtosis value exceeding the value 3 which indicates that the four stock market returns have a leptokurtic curve shape and indicates that the distribution of the data for the four stock market returns has a sharper peak around the mean. Based on the value of skewness and kurtosis, it can be shown that the return data of the four stock markets do not follow the normal distribution. To show that data is normally distributed if it has a skewness value of zero and a kurtosis value equal to 3. The identification of the extreme values in stock market returns can be seen by using a boxplot where the extreme values are at the black point in each stock market boxplot in Figure 2.



Identification of heavy tail data can be seen through the histogram in Figure 3.



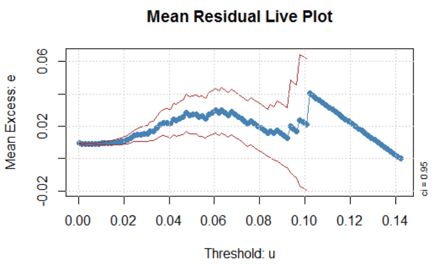
To show that the return data used are not normally distributed, it can also be done using a normal probability plot which can be seen in Figure 4.



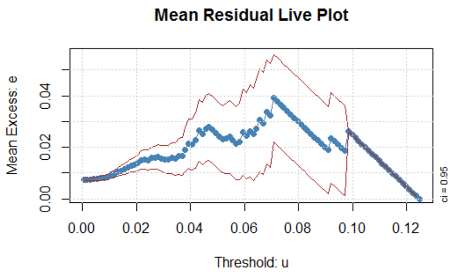
The Kolmogorov Smirnov test can also be carried out to determine whether the data used are normally distributed or not. The following statements are the results of the Kolmogorov Smirnov test on each return used in this study. The hypothesis used in the Kolmogorov Smirnov test
$H_0: F(x) = F_0(x)$ (The data follow a normal distribution),
$H_1: F(x) \neq F_0(x)$ (The data do not follow a normal distribution). The rejection area is H_0 if P-Value $< \alpha$ (5%), The results of the Kolmogorov Smirnov test can be seen in Table 2.
Table 2 Kolmogorov Smirnov Return Test Results.

Return	***P-Value***
S&P 500	2.2×10^{-16}
CAC 40	2.2×10^{-16}
DAX	2.2×10^{-16}
NIKEI 225	2.2×10^{-16}

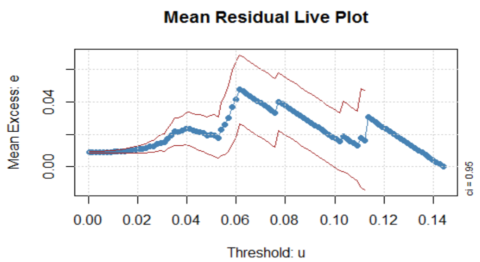
The results of the Kolmogorov Smirnov test in Table 2 have a P-Value value smaller than the value of α It can be stated that the decision to reject H_0 means that the four stock return data are not normally distributed. Determination of the threshold value can be done using the mean residual live plot (MRLP) by looking at the graphic pattern that starts to approach a linear or straight line. The MRLP for the four stock market indexes can be seen in Figure 5.



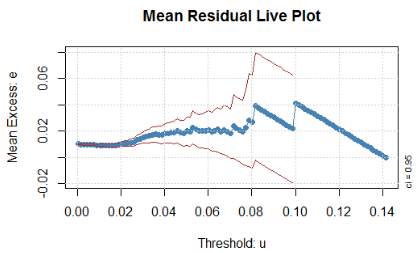
S&P 500



CAC 40



DAX



Nikei 225

In Figure 5, the mean residual live plot of the S&P 500 is starting to approach a linear or straight line, namely the threshold value (u) = 0.01 and the extreme data that is above the threshold value is 508 data. The mean residual live plot of DAX begins to approach a linear line or a straight line, namely the threshold value (u) = 0.02. with extreme data that passes the threshold value of 208 data, then the Mean residual live plot CAC 40 begins to approach a linear or straight line, namely the threshold value (u) = 0.02 with extreme data that passes the threshold value of 209 data. The mean residual of the Nikei 225 live plot starts to approach a linear line or a straight line, namely the threshold value (u) = 0.02. with 264 extreme data that passed the threshold value.
The Peak Over Threshold method applies the Pickands (1975), Balkema, and de Haan (1974) theorem which states that the sample data for extreme values taken from the Peak Over Threshold method will follow the Generalized Pareto Distribution (GPD) distribution. using the Kolmogorov Smirnov test. The results of the Kolmogorov Smirnov test on extreme data from each stock used can be seen in Table 3.

Table 3 Kolmogorov Smirnov Return Test Results

Peal Over Threshold	**P-Value**
S&P 500	0.265919
CAC 40	0.879820
DAX	0.417597
NIKEI 225	0.444970

Based on Table 3 the results of the distribution suitability test using the Kolmogorov Smirnov test, the P-Value value α > (5%) means that the data on the extreme returns of the four stock markets follow the GPD distribution.

Peak Over Threshold Parameter Estimation

The parameter estimation results using the Peak Over Threshold model can be seen in Table 4 below:

Table 4 Peak Over Threshold Parameter Estimation

Parameter	S&P 500	CAC 40	DAX	Nikei 225
Threshold (u)	0,01	0,02	0,02	0,02
n	3850	3850	3850	3850
Nu	508	208	209	264
Bentuk (ξ)	0,3318340	0,3481940	0,2686028	0,260256
Skala (σ)	0,0059274	0,0073845	0,0078308	0,007494

The parameter estimation used is the maximum likelihood parameter estimate. In Table 4 the shape parameter ($\hat{\xi}$) obtained is $\hat{\xi} > 0$ which means the heavier the tail distribution (heavy tail) while the scale parameter ($\hat{\sigma}$) shows that the variation is towards these extreme values. The four stock markets obtained the value of the shape parameter ($\hat{\xi}$) the highest is the CAC 40 stock market with an estimated shape parameter ($\hat{\xi}$) of 0.3481940 which means the heavier tail of the distribution (heavy tail) than other stock markets.

Estimated Value at Risk POT

Based on the parameter estimation obtained with the Peak Over Threshold model, the results of the POT model value risk estimation are as follows

Table 5 Estimated Value at Risk POT

Data	α		
	0.90	0.95	0.99
S&P 500	0.0117213	0.01678589	0.03418411
CAC 40	0.0159077	0.02057965	0.03695019
DAX	0.0155996	0.02065116	0.03676965
Nikei 225	0.0168316	0.02189771	0.03786507

The VaR value with the Peak Over Threshold approach for the S&P 500 stock market with a confidence level of 0.90 is 0.01172126 which means that it is possible that one day in the future the S&P 500 stock market if an investor invests Rp100.000.000,00 will experience a maximum loss of Rp. Rp1.172.126,00 or 1.172126% and with a confidence level of 0.95 it will experience a maximum loss of 2.057965% and with a confidence level of 0.99 it will experience a maximum loss of 3.41841%. In the four stock markets, the highest maximum loss value with a confidence level of 0.99 is the Nikei stock market with an estimated VaR value of 3.786507%, which means that the maximum loss for the next one day is Rp3.786.507,00.

Estimated Tail Value at Risk with POT

Based on the parameter estimation obtained with the Peak Over Threshold model, the results of the estimated Tail Value at Risk POT model are as follows:

Table 6 Estimated Tail Value at Risk POT

Data	α		
	0.90	0.95	0.99
S&P 500	0.02144726	0.02902715	0.05506593
CAC 40	0.02505082	0.03221860	0.05733425
DAX	0.02467433	0.03159697	0.05363492
Nikei 225	0.02584745	0.03269592	0.05428089

The TVaR value with a peak over threshold approach for the S&P 500 stock market with a confidence level of 0.90 is 0.02144726, which means that it is possible that one day in the future the S&P 500 stock market if an investor invests Rp100.000.000,00 will experience a maximum loss of Rp2.144.726,00 or as much as 2.144726% and with a confidence level of 0.95 it will experience a maximum loss of 2.902715% and with a confidence level of 0.99 it will experience a maximum loss of 5.506593%. In the four stock markets the highest maximum loss value with a confidence level of 0.95 is the Nikei 225 stock market with an estimated TVaR value of 3.269592% which means that the maximum loss for the next one day is Rp3.269.592,00.

Estimated Adjusted Tail Value at Risk with POT

Based on the parameter estimation obtained with the Peak Over Threshold model, the results of the estimated risk value of Adjusted Tail Value at Risk POT model are as follows:

Table 4.7 Estimated Adjusted Tail Value at Risk POT

α	c	Data			
		S&P 500	CAC 40	DAX	Nikei 225
0.90	0.01	0.01469672	0.01881819	0.01792352	0.01908968
	0.05	0.01539684	0.01947084	0.01860175	0.01976616
	0.10	0.01630263	0.02031664	0.01947347	0.02063488
0.95	0.01	0.02059621	0.02434627	0.02352545	0.02466249
	0.05	0.02175063	0.02543505	0.02459440	0.02572234
	0.10	0.02325973	0.02686148	0.02597988	0.02709449
0.99	0.01	0.04094394	0.04379842	0.04141650	0.04228168
	0.05	0.04402035	0.04677971	0.04398158	0.04478998

The value of Adj-TVaR with a peak over threshold approach for the S&P 500 stock market with a confidence level (α) of 0.90 and with the chosen constant (c) being 0.01, 0.05, 0.10, the estimated value will be greater if the value of c gets bigger. The confidence level is 0.90 with the chosen c value of 0.01, which means it is possible that one day in the future the S&P 500 stock market if an investor invests Rp100.000.000,00 will experience a maximum loss of Rp1.469.672,00 or 1.469672% and with a chosen c value of 0.05 it will experience a maximum loss of 1.539684% and with a chosen c value of 0.10 it will experience a maximum loss of 1.630263%. In the four stock markets, the highest maximum loss value with a confidence level of 0.99 and a c value of 0.05 is the CAC 40 stock market with an estimated Adj-TVaR value of 4.677971% which means that the maximum loss for the next one day is Rp4.677.971,00. the Adj-TVaR value with a peak over threshold approach using a confidence level (α=0.90,0.95,0.99) and each different constant (c= 0.01,0.05, and 0.1) provides information that the higher the level of confidence and the chosen constant, the higher the value of Adj-TVaR which represents that the potential loss will be higher.

CONCLUSIONS AND SUGGESTIONS

In this study, estimates of the risk measures of VaR, TVaR, and Adj-TVaR were obtained using the Peak Over Threshold approach. The estimated value of VaR < Adj-TVaR < TVaR is obtained on the four stock market indexes. Based on the nature of the risk measure that a good risk measure is a coherent risk measure. The characteristics of the risk measures TVaR and Adj-TVaR are sub-additive and with the same level of significance, the value of Adj-TVaR < TVaR is obtained. This indicates that the risk measure Adj-TVaR is more efficient to use in predicting the risk value of the four stocks market index. As for suggestions for further research, it is suggested to use the Block Maxima approach for extreme data and use other risk measures.

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