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Numerical Analysis of Double Integral of Trigonometric Function Using *Romberg* Method

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Abstract
In general, solving the two-fold integral of trigonometric functions is not easy to do analytically. Therefore, we need a numerical method to get the solution. Numerical methods can only provide solutions that approach true value. Thus, a numerical solution is also called a close solution. However, we can determine the difference between the two (errors) as small as possible. Numerical settlement is done by consecutive estimates (iteration method). The numerical method used in this study is the Romberg method. Romberg's integration method is based on Richardson's extrapolation expansion, so that there is a calculation of the integration of functions in two estimating ways I (h1) and I (h2) resulting in an error order on the result of the completion increasing by two, so it needs to be reviewed briefly about how the accuracy of the method. The results of this study indicate that the level of accuracy of the Romberg method to the analytical method (exact) will give the same value, after being used in several simulations.

Keywords: *Numerical Methods; Integral; Romberg's Method; Accuracy; Iteration.*

Introduction

Integral applications are found in many fields of science and engineering, such as calculating the velocity equation and measuring solar heat flux. These examples generally have complicated functions that are difficult to integrate analytically. In such a case, the settlement can actually be sought by numerical methods, where the use of the method produces a near solution that is not exactly the same as the true solution. However, we can determine the difference between the two (errors) as small as possible.

Numerical methods are techniques used to formulate mathematical problems so that they can be solved by arithmetic or arithmetic operations (addition, subtraction, multiplication, and division) (Munir, 2003).

There are several kinds of integral solutions with numerical methods such as the trapezoidal method, simpson, quadratus gauss and other methods of a higher degree (based on the Newtonian interpolation polynomial) studied in manuals such as in numerical methods and numerical analysis. However, folding integral solving techniques using numerical methods are rarely found and explained in detail. Therefore, the authors are interested in researching the completion of the folding integral, especially the two-fold integral.

Trigonometric function is one of the functions that is difficult to solve analytically using double integrals. In terms of determining the exact value of the integral double fold trigonometry functions sometimes have difficulty even using several integration methods, for example the substitution method, partial method or other methods. This is due to the many variations or shapes that can occur from a trigonometric function.

The integration method used by the writer to solve the doubling integral is the Romberg integration method. This is based on the acquisition of integration values which is more accurate when compared to other integration methods. Romberg integration is a method of improvement of numerical integration methods. This is based on the truncation error of the trapezoid method whose magnitude is almost proportional to the square of bias width *(h2) I*.

Romberg's integration is based on Richardson's extrapolation expansion, so that there is a calculation of the integration of functions in two estimating ways I (h1) and I (h2) which results in an error order on the result of completion increasing by two. If the error order increases, the error value decreases, and if the error value decreases, the numeric integration value will give a value that is close to or equal to the exact value. Based on this, the hope of the writer by using Romberg integration in solving the double integrals in this research is that Romberg integration is able to minimize the calculation error and allow it to give results that are close to the exact value (true value).

Method

This research is a pure research (theoretical study), which aims to analyze the integrals of trigonometric functions using the Romberg method by reviewing some literature relating to numerical analysis, calculus, numerical methods, articles and others in solving doubling integrals conducted in the Department of Mathematics UNM and carried out to completion.

The working procedure adopted in this study is to take a number of examples of trigonometric functions to be carried out by doubling integrals both by the exact method and numerically. Next, analyze the accuracy of the results of the completion of the two-fold Romberg method against the exact method values in the example.

The exact double integral solving algorithm is as follows:

1. Define functions and integral boundaries.
2. Select the integral limit to be solved first (x or y).
3. Determine the completion of the first integral analytically.
4. The first integral result (I_1) is analytically reintegrated.
5. Double integral results (I_2) with analytic methods.

The two fold integral solving algorithm with the Romberg method is as follows:

1. Define functions and integral boundaries.
2. Select the integral limit to be solved first (x or y).
3. If the selected limit is x counted:
$$h = x_2 - x_1$$
If the selected limit is y counted
$$h = y_2 - y_1$$
4. If the selected limit is x counted
$$R(1,1) = T_0 = \frac{h}{2}\left(f(x_1,y) + f(x_2,y)\right)$$
If the selected limit is y counted
$$R(1,1) = T_0 = \frac{h}{2}\left(f(x,y_1) + f(x,y_2)\right)$$
5. If the selected limit is x counted:
$$R(j,1) = T_{n+1}$$
$$= \frac{T_n}{2} + \frac{h}{2^{n+1}}\sum_{r=1}^{2^n} f_{2r-1}$$
where
$$f_i = f\left(x_1 + i\frac{h}{2^{n+1}}\right),$$

$n = 0,1,2, \dots\ dan\ f_{2r-1} = f_i$

If the selected limit is y counted:

$$R(j,1) = T_{n+1}$$
$$= \frac{T_n}{2} + \frac{h}{2^{n+1}} \sum_{r=1}^{2^n} f_{2r-1}$$

where

$f_i = \left(y_1 + i.\frac{h}{2^{n+1}}\right),$

$n = 0,1,2, \dots,$

$f_{2r-1} = f_i$

6. The next count with Romberg's integration with Richardson extrapolation is

$$R(j,k) = \frac{4^{k-1}R(j.k-1) - R(j-1,k-1)}{4^{k-1}-1}$$

for $2 \le k \le j$, where the initial value is a trapezoidal quadrat

$$R(1,1) = T_0 = \frac{b-a}{2}\left(f(a) + f(b)\right)$$

7. Check whether $\left|I_1 - \hat{I}_{n,n}\right| < \varepsilon$? if not, continue to the next iteration, $\ddot{I}_{n+1,n+1}$, if yes, then the iteration calculation is stopped.
8. The results of the first integration (6) are integrated again, i.e. return to steps (3) to (6).
9. Check whether $\left|I_2 - \hat{I}_{n,n}\right| < \varepsilon$? if not, continue to the next iteration, $\ddot{I}_{n+1,n+1}$, if yes, then the iteration calculation is stopped.

Results and Discussion

Results

Romberg's integration method is based on Richardson's extrapolation expansion, so that there is a calculation of the integration of functions in two estimating ways I (h1) and I (h2) which results in an error order on the result of the completion increasing by two.

The example problem used is the two-fold trigonometric function.

$$\iint_D e^{x+y} \cos(x) \sin(y)\ dA\ ,$$
$$D = \{(x,y) | 0 \le x \le \pi, 0 \le y \le \pi\}.$$

The result error of integral two function from $e^{x+y} \cos(x) \sin(y)$ with method Romberg to eksak value -145,693253 dengan $\varepsilon = 0,000001$

Iterasi	metode 1	Method 2 (Simpson)	Method 3 (Boole)	Repair 4	Repair 5

	(Trapezium)				
1	145,693253				
2	54.486337	24.084031			
3	14.661742	1.386877	0.126267		
4	3.725229	0.079725	0.007418	0.005531	
5	0,934947	0,004853	0.000138	0.000022	0.000000

Discussion

Analytically solving the first Integral value (I_1) in the x direction is obtained $-12{,}070346\, e^y \sin(y)$. Next will be calculated the two-fold integral of the direction. In the same way in the first step, the second Integral value is obtained (I_2) in the y direction is $-145{,}693253$
So it's integral,

$$\iint_D e^{x+y} \cos(x) \sin(y)\ dA \ ,$$
$$D = \{(x,y) | 0 \le x \le \pi, 0 \le y \le \pi\}.$$

using analytic methods is $-145{,}693253$.

Next, the doubled integral settlement will be done in the example problem using the Romberg method. The numerical approach will use the tolerance value as the reference limit for the iteration process. Tolerance value ε that where used is $\varepsilon = 0{,}000001$. That is, iteration will stop if the error value is less than ε, or

$$\left|I_i - \hat{I}_{j,k}\right| < \varepsilon.$$

The first integral result, in the direction x, obtained the error value

$$\left|I_1 - \hat{I}_{5,5}\right| = |(-12{,}070346\, e^y \sin(y)) - (12{,}070346\, e^y \sin(y))|$$

$$= 0.000000\, e^y \sin(y) < \varepsilon$$

then the iteration is stopped. The number of iterations in the first integral, the x direction is 5 iterations. Next will be calculated the second integral, the y direction.
The second integral result, in the y direction, the error value is obtained

$$\left|I_2 - \hat{I}_{5,5}\right| = |(-145{,}693253) - (-145{,}693253)|$$

$$= 0.000000$$

Because $\left|I_1 - \hat{I}_{5,5}\right| = 0.000000 < \varepsilon$ then the iteration is stopped. So it's integral $e^{x+y}\cos(x)\sin(y), \{(x,y)|0 \le x \le \pi, 0 \le y \le \pi\}$ using the Romberg method is $-145{,}693253$.

From the result of reseach show that the two-fold integral is a function $e^{x+y}\cos(x)\sin(y)$ using the Romberg method is $-145{,}693253$. The number of iterations in the first integral, the x direction, is 5 iterations. The solution obtained using the Romberg method is the same as the exact (analytic) solution shown in the fifth iteration process, namely the fifth (improvement) method. indicates the error value for the Romberg method. The error for each method in the fifth iteration is always greater than the error for the next (fix) method. In method 1 (Trapezoid) gives an error value of $0{,}934947$, in method 2 (Simpson) gives an error value of, in method 3 (Boole) gives an error value of $0{,}000138$, in method (repair) 4 gives an error value of $0{,}000022$, in method (repair) 5 gives an error value of $0{,}000000$. This shows that the level of accuracy of the Romberg method settlement converges to the exact settlement.

Conclusions and suggestions

Based on the simulation example that has been done in solving the double integrals using the Romberg method it is found that the results of the first method to the next method, the solution given always approaches the exact value. That is, the level of accuracy of the Romberg method to the analytical method (exact) will give the same value. While the error value gets smaller each additional iteration. This shows the convergent error value.

The advantage of the Romberg method is that the Romberg integration method can be used to determine folding integral solutions that are difficult or even cannot be solved analytically. However, the Romberg method requires a fairly long iteration process. And it is not simple to track the process for convergence, and in the calculation there is a high probability that the process will provide divergent results, unless the tolerance value (error) given is quite precise (small).

After carrying out and seeing the research results obtained, the following suggestions can be presented:

1. In an effort to increase the level of accuracy required the application of several examples of trigonometric functions.
2. Because the numerical method uses several iterative processes, it is very much needed in the next research program or application for doubling integral calculations with the Romberg method.

Reference

Chapra, Steven C. & Chanale, Raymond P. 1991. *Metode Numerik untuk Teknik*. Jakarta: UI-press.

Manalaksak, D.G. 2004. *Fungsi Berdasarkan Filsafat Matematika*. Desember 30, 2012. http://jurnal.filsafat.ugm.ac.id/index.php/PDFInterstitial/54/52

Mathews, J.H. 1987. *Numerical Methods Using Matlab Fourth Edition*. Amerika: Pearson Prentice Hall.

Munir, R. 2003. *Metode Numerik*. Bandung: Informatika Bandung.

Mursita, D. 2008. *Integral Fungsi Trigonometri*. Desember, 30, 2012. www.geocities.ws/dmursita/matdas/v-3.pdf

Nasution, A. 2001. *Metode Numerik dalam Ilmu Rekayasa Sipil*. ITB Bandung: Bandung.

Parcel, Varberg, Rigdon. 2010. *Kalkulus edisi kesembilan*. Jakarta:Erlangga

Sahid. 2004. *Pengantar Komputasi Numerik dengan MATLAB*. Yogyakarta: Penerbit ANDI Yogyakarta.

Triatmodjo, B. 2002. *Metode Numerik Dilengkapi dengan Program Komputer*. Yogyakarta: Beta Offset.

Weber, Jean E. 1999. *Analisis Matematik Penerapan Bisnis dan Ekonomi Edisi Ke empat*.

Jakarta: Erlangga.